ACCOMMODATING MONETARY POLICY AND THE LIQUIDITY EFFECT

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The relation between the existence of a liquidity effect and the institutional arrangements that coordinate fiscal and monetary policies and preserve the government's intertemporal budget balance is explored in the context of a simple dynamic, general equilibrium macroeconomic model.
The model simulations show that a regime of fiscal dominance and monetary accommodation can explain the occurrence of liquidity effects as a result of monetary shocks.
The source of the liquidity effect in the model is the fact that unpredictable open market operations performed by the monetary authority (i.e., monetary policy shocks) are followed by open market operations of the opposite sign in order to preserve the government's intertemporal balance. Hence, an expansive monetary shock announces restrictive open market operations in the future that cause a drop in both the expected rate of inflation and the nominal interest rate.

1 INTRODUCTION

The liquidity effect is the "purported statistical relation between expansion of bank reserves or monetary aggregates (or perhaps only surprise expansions of these aggregates) and short run reductions in the short-term interest rates."[1] The existence of such relation is usually taken for granted in both academic and policy discussions about the monetary transmission mechanism. However, the evidence supporting the liquidity effect is rather controversial.[2] Moreover, there is no consensus on the theoretical explanation for the liquidity effect.[3]

For instance, while the traditional IS-LM models with sticky prices and without rational expectations clearly exhibit a liquidity effect, the introduction of rational expectations in those models produces an ambiguous result. The reason is that an expansionary monetary shock causes the price level to increase in the long run; since the price level is fixed in the short run, expected inflation clearly rises. On the other hand, the real interest rate drops in the short run because the equilibrium in the goods market requires a lower real interest rate for the given increase in money real balances and in output. Consequently, the total effect in the market interest rate is ambiguous and the negative statistical relation between monetary aggregates and market interest rates does not need to be present.[4]

Sticky-prices models in which agents explicitly maximize intertemporal utility present basically the same problem: an expansionary monetary shock causes an increase in expected inflation that offsets the decrease in the real interest rate and the total effect is ambiguous in general. Ohanian and Stockman [14] find that, for sensible calibrations of parameterized models, the expected inflation effect is dominant in one sector models, while the real interest rate drop is dominant in two sector models.

Among the general equilibrium (flexible prices) macroeconomic models, it is usually even harder to obtain a liquidity effect. The reason is that, for positively serially correlated monetary aggregates, a current increase in the money supply announces further increases in the future and, consequently, higher future inflation. Since price flexibility does not allow for the real

[1] Ohanian and Stockman ([14], p. 3.)

[2] See Pagan and Robertson [15] and [16].

[3] See Ohanian and Stockman [14] and Hoover [9].

[4] Hoover [9] gives a detailed explanation of the workings of the liquidity effect in the IS-LM model.

effects that occur in sticky-prices models, the expected inflation effect works basically alone and causes the nominal interest rate to increase after a positive monetary shock. An important exception to this generalization is the class of limited participation models[5], in which the restrictions imposed on financial transactions for some agents allow monetary shocks to affect the real interest rate and offset the expected inflation effect. Again, the net effect on the market nominal interest rate is ambiguous, but for some calibrations a liquidity effect is generated. The main objection to this type of models is that the restrictions imposed on financial transactions seem to be rather artificial.

This paper explores the plausibility of an alternative explanation for the liquidity effect in the context of a dynamic, general equilibrium macroeconomic model with flexible prices. The basic idea is that a monetary policy shock announces future open market operations of the opposite sign as a consequence of the fiscal implications of the initial shock. For example, an expansive monetary policy shock (i.e., an unexpected open market operation that increases the nominal money supply and decreases the nominal value of the government's outstanding debt) reduces the interest payments on the public debt. Assuming that a relevant intertemporal budget constraint applies for the government (consolidating the fiscal and monetary branches together), the effect of the shock must be offset by future fiscal and/or monetary actions (i.e., greater future primary deficits or less future seigniorage, respectively). If the policy regime is such that the main adjustment effort corresponds to the monetary branch of the government, then the initial shock by itself announces less future seigniorage, lower future rate of growth of the money supply, and lower future inflation. This drop in the expected rate of inflation causes a current drop in the nominal interest rate, materializing a liquidity effect.

The liquidity effect in this paper is generated by a model that shares the spirit of the "unpleasant monetarist arithmetic" by Sargent and Wallace [17]. The policy setup is such that the fiscal authority dominates or "plays first" (in the sense that the stochastic process for the primary fiscal deficit is independent of the actions of the monetary authority), and the monetary authority necessarily accommodates the process for seigniorage-revenue creation in order to preserve the government's intertemporal balance. The same

[5] See, for example, Christiano [4], Christiano and Eichenbaum [5], Fuerst [8], and Lucas [13].

idea drives the results presented in Leeper [11] and Leeper [12]. In Leeper's terminology, the policy setup that generates a liquidity effect in this paper is one of "active fiscal policy" and "passive monetary policy".

More generally, the results in the papers by Sargent and Wallace and by Leeper mentioned above, as well as the results in this paper, can be regarded as particular cases of the mechanism emphasized by the literature on the 'fiscal theory of the price level determination'.[6] This literature formalizes the idea that the price level and the inflation rate are ultimately determined by the total amount of nominal liabilities of the public sector, rather than by the money supply only (as the quantitative theory of money predicts). The link between the total amount of government's nominal liabilities and the price level is the government's intertemporal budget constraint (as well as its counterpart, namely the representative agent's intertemporal budget constraint). Whenever there is a mismatch between the government's amount of nominal liabilities and the present value of its future primary surpluses, a change in the price level is required in order to preserve the market clearing feature of all markets. The most striking result of the fiscal theory of the price level determination is that the price level and the inflation rate can change (for some well defined policy experiments) without any change whatsoever in the money supply (what is required is an appropriate change in the supply of public nominal bonds).[7] This result (whose empirical relevance is questionable) goes beyond the scope of the previous literature that emphasized the importance of the government's intertemporal budget constraint (Sargent and Wallace, and Leeper), since in this literature the price level and the monetary aggregates are highly correlated in the long run, either because the monetary authority dominates the policy game or because it accommodates its policy to the requirements of the government's intertemporal balance. In this context, this paper can be better identified with the previous literature on fiscal/monetary dominance/accommodation than with the more general literature on the fiscal theory of price level determination.

The paper is organized as follows: section 2 presents the basic model and its main results, section 3 extends the analysis to alternative specifications of the fiscal and monetary policy institutions, and section 4 concludes.

[6] Some references in this literature are the following: Auernheimer and Contreras [1], Coleman [6], Kocherlakota and Phelan [10], Sims [18], Woodford [19], Woodford [20].

[7] See Woodford [20], p. 20-23.

2 THE BASIC MODEL

The following model corresponds to a simple endowment, monetary economy. There are two sectors: the household and the government. We now proceed to examine the main features of each sector.

2.1 The Household

There is a continuum of infinitely-lived households. Each household has a fixed endowment of y units per period of the unique, perishable consumption good. The household also has an initial stock of nominal money m_0 and an initial stock of nominal public debt b_0. The household also gets a nominal transfer f_t per period from the government. Each period, the household observes the state of the economy and decides how to allocate its wealth between consumption and asset accumulation (money, bonds, and, in principle, contingent claims). Money is issued by the government only.

2.1.1 Household's Problem

The household solves the following problem:

$$\max_{\{c_t, m_{t+1}, b_{t+1}, h(s_{t+1})\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \log\left(\frac{m_{t+1}}{P_t}\right) \right] \tag{1}$$

subject to:

$$c_t + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{P_t} + \int r(s_{t+1}|s_t) \cdot h(s_{t+1}) ds_{t+1} =$$

$$y + \frac{m_t}{P_t} + (1 + i_t) \frac{b_t}{P_t} + \frac{f_t}{P_t} + h(s_t); \tag{2}$$

and

$$\frac{(1 + i_0 (s_{-1})) \cdot b_0 (s_{-1})}{P_0 (s_0)} + h(s_0) =$$

$$\sum_{t=0}\int R(s_t|s_0)\cdot\left[c_t(s_t)+\left(\frac{m_{t+1}(s_t)-m_t(s_{t-1})}{P_t(s_t)}\right)-y-\frac{f_t(s_t)}{P_t(s_t)}\right]\cdot ds_t \quad (3)$$

where:

$$R(s_t|s_0)=\prod_{j=0} r(s_{j+1}|s_j) \quad (4)$$

initial conditions for assets are given;
$c_t, m_t \geq 0, \forall t$;
i_0 is given;
$h(s_0) = 0$;
current prices are taken as given;
random processes are known;
future prices are rationally expected.

Equation (2) is the period budget constraint. Equation (3) is the intertemporal, stochastic budget constraint that prevents Ponzi schemes.

2.1.2 Some Definitions and Conventions

c is consumption; $\frac{m_{t+1}}{P_t}$ is the household's real currency balance; $\frac{b_{t+1}}{P_t}$ is the household's real stock of public debt; $\frac{f}{P}$ is a real lump-sum transfer from the government (which, in principle, can be positive, negative, or zero); i is the nominal interest rate on public bonds; P is the price level. s_t is the state of nature in period t; $h(s_t)$ is the number of contingent claims that payoff one unit of the consumption good in period t if and only if the state of nature s_t occurs at period t; $r(s_{t+1}|s_t)$ is the price in period t (when state s_t is observed) of a contingent claim that pays one unit of the consumption good in period $t+1$ if and only if the state of nature s_{t+1} occurs in period $t+1$.

In period t, after observing all period t shocks, the agent chooses the $t+1$ value of each asset.

Interest rates indexed t clear the financial assets markets in period $t-1$ and must be paid in period t, and those indexed $t+1$ clear the markets in period t and must be paid in period $t+1$.

2.2 The Government

2.2.1 Budget Constraints

The period budget constraint of the government (in nominal and per capita terms) is the following:

$$F_t + (1 + i_t)B_t = (M_{t+1} - M_t) + B_{t+1} \tag{5}$$

F_t represents the primary fiscal deficit (lump-sum transfers to the household minus lump-sum taxes[8]); B_t is the nominal public debt issued in period $t-1$ that must be paid (along with the corresponding interest) in period t; M_t is determined in period $t-1$ and taken as given in period t.

Prevention of Ponzi schemes and a full use of resources on the part of the government require the following stochastic intertemporal budget constraint (in real and per capita terms) to hold:

$$\frac{(1 + i_0(s_{-1})) \cdot B_0(s_{-1})}{P_0(s_{-1})} =$$

$$\sum_{t=0}^{\infty} \int R(s_t|s_0) \cdot \left[\left(\frac{M_{t+1}(s_t) - M_t(s_{t-1})}{P_t(s_t)} \right) - \frac{F_t(s_t)}{P_t(s_t)} \right] \cdot ds_t \tag{6}$$

2.2.2 Behavioral Assumptions

The ratio of the fiscal primary deficit to last period's money stock ($\frac{F_t}{M_t}$) is assumed to be governed by the following law of motion:

$$\left(\frac{F_t}{M_t} \right) = \gamma_0 + \gamma_1 \left(\frac{F_{t-1}}{M_{t-1}} \right) + u_{1,t} \tag{7}$$

$$u_{1,t} \sim N(0, \sigma^2_{u_1}) \tag{8}$$

where γ_0 and γ_1 are constants and $\gamma_1 \in (0, 1)$.

In addition, the debt-to-money ratio obeys the following law of motion:

$$\left(\frac{B_{t+1}}{M_{t+1}} \right) = \gamma_2 + \gamma_3 \left(\frac{B_t}{M_t} \right) + u_{2,t} \tag{9}$$

[8] There are no public expenditures in the model.

$$u_{2,t} \sim N(0, \sigma^2_{u2}) \tag{10}$$

where γ_2 and γ_3 are constants and $\gamma_3 \in (0,1)$.

The variable $\frac{F}{M}$ represents fiscal policy in this model. F is a per capita lump-sum transfer to the household minus any per capita lump-sum tax. Since there is no government expenditure in the model, F represents the nominal primary fiscal deficit. The basic idea is that fiscal policy is completely exogenous, so that monetary policy cannot involve any commitment about present or future lump-sum taxes or transfers to the household. However, fiscal policy does affect monetary policy, since present and future fiscal primary deficits must be eventually paid for with seigniorage revenue, while present or future fiscal primary surpluses decrease the government's need for present or future seigniorage.

On the other hand, the $\frac{B_{t+1}}{M_{t+1}}$ ratio represents monetary policy in the model, which is accomplished by means of open market operations. Since, as was said before, monetary policy cannot affect present or future primary fiscal deficits, but it does affect total fiscal deficits because it affects the amount of interest payments to the household, then it is clear that monetary shocks (i.e., shocks to the $\frac{B}{M}$ ratio) must be eventually offset by open market operations of opposite sign if the government's intertemporal budget constraint is to hold.[9] That is why the $\frac{B}{M}$ ratio is modeled in the form of a stationary autoregressive process. A positive shock to this process is an unpredictable open market operation in which the amount of public debt in the private sector's hands increases as the outstanding money supply decreases. A negative shock is an unpredictable open market operation in the opposite direction.

Together, these two policy variables determine the rate of growth of the money stock in a period by period basis, as required by the government's period budget constraint. The gross rate of growth of the money stock can be determined by dividing the period budget constraint (5) over M_t, and solving for $\frac{M_{t+1}}{M_t}$. The result is:

$$\frac{M_{t+1}}{M_t} = \frac{1 + \left(\frac{F_t}{M_t}\right) + (1+i_t)\left(\frac{B_t}{M_t}\right)}{1 + \frac{B_{t+1}}{M_{t+1}}} \tag{11}$$

[9] Changes in the price level (that affect the real value of the government's nominal liabilities) are also required, in general, to guarantee that the government's intertemporal balance holds.

We now elaborate on the implications of our modeling strategy for the public sector. Using the insights of Sargent and Wallace [17] and Leeper [11], it is clear that fiscal and monetary policies must be somehow coordinated if the intertemporal budget constraint of the government (6) is to hold. In this model, the stationarity of the $\frac{B}{M}$ ratio implies that monetary policy bears the burden of returning the $\frac{B}{M}$ ratio to its steady state value.

The crucial feature of the above specification for the policy variables is that a monetary policy shock (i.e., an open market operation that causes an unpredictable change in the debt-to-money ratio) is followed by a tendency to return of the debt-to-money ratio to its unconditional mean. This result is achieved by the normal workings of monetary policy (open market operations in opposite direction to the shock), and generates a liquidity effect in the model economy. For example, if the economy is at its nonstochastic steady state, then an expansive monetary shock that causes the $\frac{B}{M}$ ratio to drop is followed by open market operations that increase the amount of outstanding public debt and decrease the stock of money in public's hands until the $\frac{B}{M}$ ratio returns to its steady state level. Since it is known that this return to the steady state requires that the monetary authority engages in restrictive open market operations, then the market anticipates a reduction in inflation that drives down the nominal interest rate. This is the nature of the liquidity effect displayed by the model economy.

2.3 Calibration

There are eight parameters in the model. Two of them (β, y) correspond to the household sector, and their values were assigned in order to make the model 'sensible' in some broad sense, rather than to match some specific features of a real world economy. In particular, the value for β (the discount factor) was equated to that in Castañeda [3], and the value for y (output endowment) was equated to the steady state value of output in Castañeda [3].[10]

The rest of the parameters $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \sigma_{u_1}, \sigma_{u_2})$ correspond to the public sector. For simplicity, γ_0 was arbitrarily set to zero (so that the non-stochastic steady state value of the fiscal primary deficit be zero). γ_1, γ_2, γ_3, σ_{u_1}, and σ_{u_2} were taken from Castañeda [2], who estimated them from

[10] Castañeda [3] calibrates an economy similar to the one in this paper (but more involved) to the quarterly US data.

the US data.[11]

The parameter values are the following:

$\beta = 0.984$
$y = 1.47$
$\gamma_0 = 0$
$\gamma_1 = 0.932$
$\gamma_2 = 0.008$
$\gamma_3 = 0.998$
$\sigma_{u_1} = 0.029$
$\sigma_{u_2} = 0.115$

2.4 Solution

Some aspects of the model are solved trivially. Firstly, market clearing requires that $c_t = y$ for all t. Secondly, since all households are homogeneous and the government does not issue or hold any contingent claim, then the net supply of contingent claims equals zero for all periods and all possible states of the world.[12] In addition the first order condition of optimization for the household with respect to bonds holdings, evaluated in equilibrium, implies that the expected real interest rate is constant and equal to the discount rate:

$$E_t \left[\frac{1 + i_{t+1}}{(P_{t+1}/P_t)} \right] = \frac{1}{\beta} \tag{12}$$

The rest of the model was solved using the methods explained in Farmer ([7], Ch. 1-3); i.e., linearizing the Euler equations around the nonstochastic steady state, and solving the resulting linearized dynamic system. The stationary nature of the two government's policy variables guarantees the existence of a stationary solution to the linearized model. In this solution, the present value of the government's real debt in the very distant future converges to zero, so both the government's and the representative agent's budget constraints hold.

[11] The variables $\frac{F}{M}$ and $\frac{B}{M}$ were constructed from the published data as explained in Castañeda [2].

[12] Contingent claims were introduced in the model because their prices are required to characterize the stochastic, intertemporal budget constraint of the household, as well as that of the government.

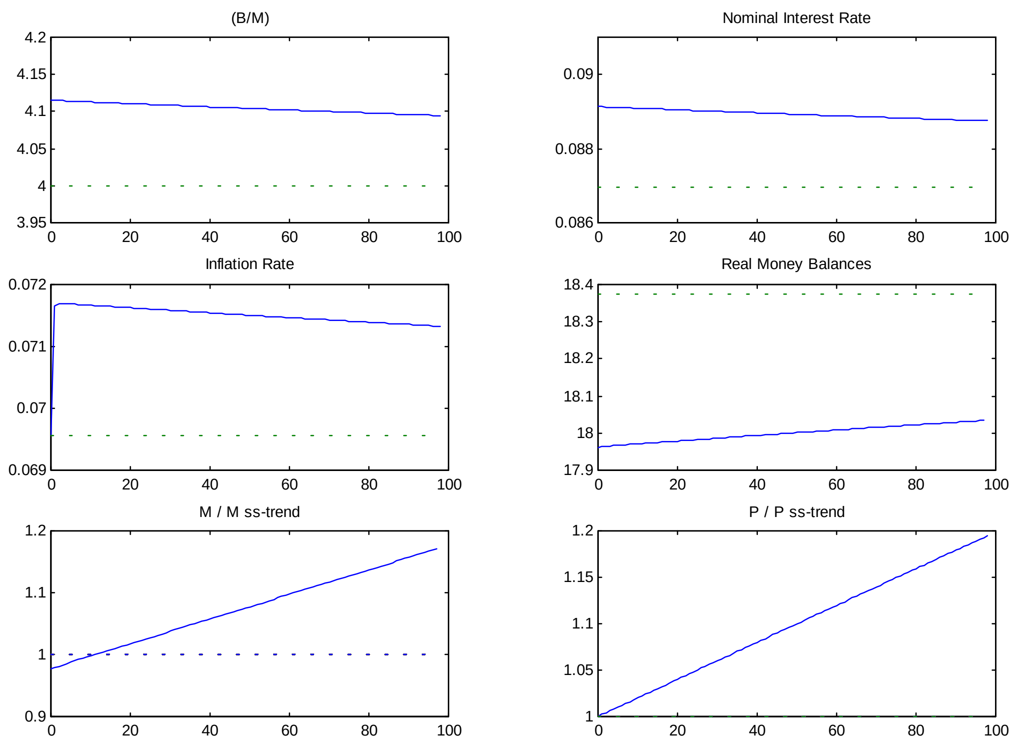

Figure 1: A restrictive monetary policy shock

The impulse-response functions in Figure 1 show the effect of a restrictive monetary policy shock. The shock consists of an unpredictable open market operation in which the monetary authority sells bonds. Consequently, the $\frac{B}{M}$ ratio increases, and it later returns (asymptotically) to its initial (steady state) value, following its law of motion. Since the increase in $\frac{B}{M}$ also increases the amount of present and future interest payments from the government to the private sector, and since present and future fiscal primary deficits are unaffected by the shock, then future seigniorage revenue must increase to preserve the original government's intertemporal balance. It is precisely the increase in future seigniorage what causes the rate of growth of the money supply to increase and the $\frac{B}{M}$ ratio to return to its initial level. Of course, this result is achieved at the cost of an increase in future inflation, which, in turn, explains the increase in the nominal interest rate (i.e., the liquidity effect). The nominal interest rate converges to its initial value as the $\frac{B}{M}$ ratio decreases and the need for future seigniorage also decreases.

Since the cost of holding real money balances increases with the nominal interest rate, those balances jump down at the moment of the shock, and then converge from below to its initial level as the nominal interest rate decreases. The nominal money balances, on the other hand, drop at the moment of the shock, and then grow at a rate greater than its steady state rate of growth; consequently, in the long run, the 'restrictive' monetary shock generates an expansive monetary policy. The price level is not affected on impact because the decrease in nominal money balances is offset by the decrease in real money balances.[13] [14] However, the price level also grows after the shock at a rate greater than its steady state rate of growth, validating the expected increase in inflation.

[13]This result is caused jointly by the specification of the period utility function (unitary elasticity of substitution between consumption and real balances) and the independence of the two policy processes. If that elasticity of substitution were lower, then real money balances would be less responsive to interest rate changes and the price level would drop on impact.

The price level would drop on impact also if fiscal policy were somehow affected by the monetary shock (see VAR case below).

[14]Leeper [11] obtains the same result in an environment in which preferences and technology are identical to those in this model, but the policy variables are specified differently.

2.5 A Fiscal Shock

Figure 2 shows the impulse-response functions corresponding to a fiscal shock. A positive fiscal shock is defined as an unpredictable increase in the $\frac{F}{M}$ ratio (i.e., an unpredictable increase in the primary fiscal deficit or an unpredictable decrease in the fiscal primary surplus). The stationary, autoregressive nature of the $\frac{F}{M}$ process implies that the increase in the deficit is persistent. Moreover, since the serial correlation for the process is positive, the fiscal shock does not announce (and the private sector does not expect) future primary surpluses (or reduced deficits) that offset the effects of the shock on the government's intertemporal balance. In present value terms, the government's balance requires the generation of additional revenue to cover the present and future increased deficits, and the only available source of revenue (other than primary surpluses) is seigniorage. Since we are assuming that the $\frac{B}{M}$ ratio remains constant, then both B and M need to grow at a rate greater than their steady state rate of growth in order to cover the deficits, until the $\frac{F}{M}$ ratio returns to its steady state level. The increased rate of growth in M causes an increase in the inflation rate and, hence, an increase in the nominal interest rate. Consequently, the real money balances remain below their steady state value during the transition. In the period of the shock, the increase in the nominal money stock and the simultaneous drop in the demand for real balances causes the price level to jump up.

3 CHARACTERIZING POLICY INSTITUTIONS

The results of the basic model presented above correspond to a very particular set of assumptions regarding the government's behavior. In particular, the assumptions that the $\frac{F}{M}$ process is unaffected by monetary policy and that the $\frac{B}{M}$ process is stationary imply that the monetary authority accommodates its behavior in the long run in order to preserve the government's intertemporal balance, as explained above. We call this institutional arrangement "fiscal dominance and monetary accommodation". The opposite case would be one of "monetary dominance and fiscal accommodation". These two polar cases can be relaxed by allowing the dominant authority's behavior to be affected by the accommodating authority's actions, even though

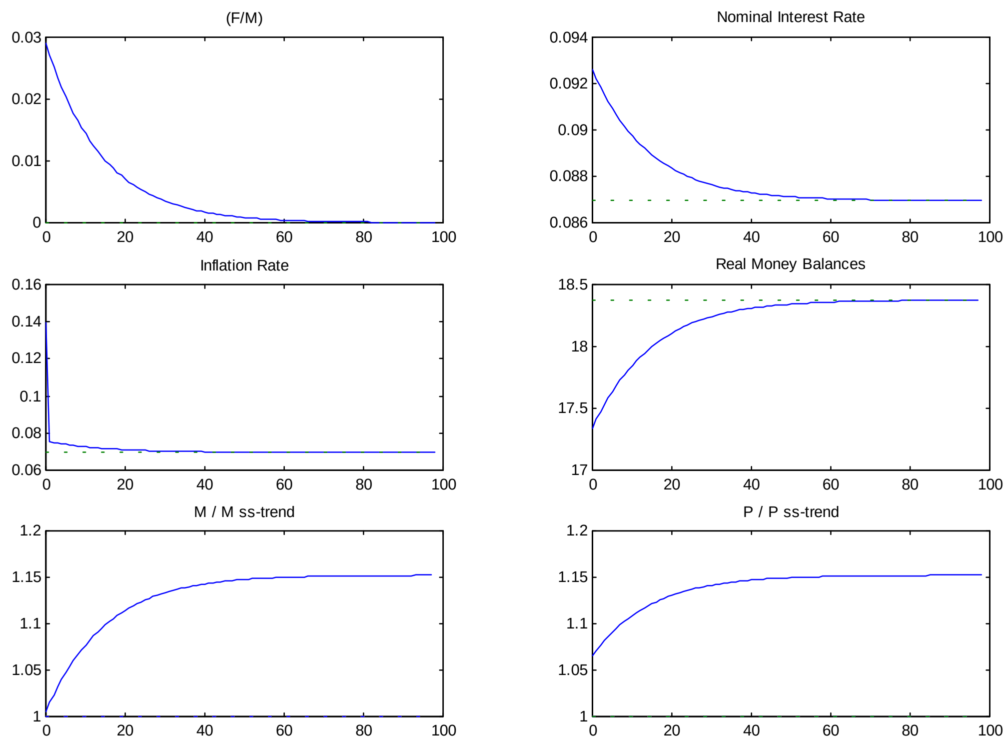

Figure 2: A positive fiscal shock

the accommodating authority has the burden of guaranteeing that the government's intertemporal balance holds in the long run. We analyze these possible cases below.

3.1 Fiscal Dominance and Monetary Accommodation: VAR Case

The only distinction between this case and the basic model above is that the two policy variables $\frac{F}{M}$ and $\frac{B}{M}$ are no longer assumed to follow independent autoregressive processes, but rather a vector autoregressive process (VAR). In particular, the model for the government's behavior is the following:

Let us define the following vectors:

$$\overrightarrow{g}_t = \left[\left(\frac{F_t}{M_t} \right), \left(\frac{B_{t+1}}{M_{t+1}} \right) \right]' \tag{13}$$

$$\overrightarrow{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{14}$$

where q_1 and q_2 are constants.

Now, we assume that the vector $\overrightarrow{g}_t$ obeys the following stationary, structural, recursive VAR process:

$$G_0 \cdot \overrightarrow{g}_t + G(Ł) \cdot \overrightarrow{g}_{t-1} = \overrightarrow{q} + \overrightarrow{u}_t \tag{15}$$

where G_0 is a lower-triangular, 2×2 matrix with ones in its diagonal; $G(Ł)$ is a matrix polynomial in the lag operator Ł; and $\overrightarrow{u}_t$ is a vector stochastic process such that:

$$\overrightarrow{u}_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \sim iid, N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{u_1} & 0 \\ 0 & \sigma^2_{u_2} \end{bmatrix} \right) \tag{16}$$

The reduced form of (15) is the following:

$$\overrightarrow{g}_t = \Gamma(\text{Ł}) \cdot \overrightarrow{g}_{t-1} + \Lambda + \overrightarrow{e}_t \tag{17}$$

where

$$\Gamma(\text{Ł}) = -G_0^{-1} \cdot G(\text{Ł}) \tag{18}$$

$$\Lambda = G_0^{-1} \cdot \overrightarrow{q} \tag{19}$$

$$\overrightarrow{e}_t = G_0^{-1} \cdot \overrightarrow{u}_t \tag{20}$$

As before, $\frac{F}{M}$ and $\frac{B}{M}$ determine together the rate of growth of the money supply in a period by period basis:

$$\frac{M_{t+1}}{M_t} = \frac{1 + \left(\frac{F_t}{M_t}\right) + (1 + i_t)\left(\frac{B_t}{M_t}\right)}{1 + \frac{B_{t+1}}{M_{t+1}}} \tag{21}$$

The fact that G_0 is a lower triangular matrix implies that the monetary authority observes the contemporary fiscal shocks before the monetary shock is realized, while the fiscal authority does not observe the contemporary monetary shock before the fiscal shock is realized. There is no strong theoretical justification for this assumption; its purpose is only to allow the estimation of the structural form of the VAR, since some sort of identifying assumption is needed in order to distinguish one structural policy shock from the other in the data. However, as long as the VAR system is stationary, a shock to the $\frac{B}{M}$ ratio will be followed by an eventual return of this ratio to its unconditional mean, no matter what the ordering of the variables is in the structural VAR; and this is all that is required for the basic results of the model to hold, as explained below.

In this model, the stationarity of the VAR process for $\overrightarrow{g}_t$ implies that both fiscal and monetary policies share the burden of returning the two policy processes (the primary fiscal deficit and the debt over money base ratio) to their unconditional means; the particular way in which this task is accomplished is determined by the VAR parameters.

As before, a monetary policy shock (i.e., an open market operation that causes an unpredictable change in the debt-to-base money ratio) is followed by a return of the debt-to-base money ratio to its unconditional mean. But now, this result is achieved by a mix of fiscal policy (altering the magnitude of the fiscal primary deficit) and monetary policy (open market operations in opposite direction to the shock). If the role of the open market operations in opposite direction to the monetary shock is important enough in the policy mix that preserves the government's intertemporal balance, then the model displays a liquidity effect. For example, if the economy is at steady state, then an expansive monetary shock that increases the stock of money in the public's hands and decreases the outstanding stock of public debt is followed by a combination of primary fiscal deficits and open market operations that increases the amount of outstanding public debt and decreases the stock of money in public's hands until both policy variables return to their steady state levels. If it is expected that the return to the steady state values requires that the monetary authority engages in restrictive open market operations, then the market anticipates a reduction in inflation that drives down the nominal interest rate. This is the nature of the liquidity effect displayed by the model.

3.1.1 Calibration

The stochastic process for the government's policy variables was approximated by the following reduced-form VAR process:

$$\overrightarrow{g}_t = \Gamma_1 \cdot \overrightarrow{g}_{t-1} + \Lambda + \overrightarrow{e}_t \tag{22}$$

where:

$$\overrightarrow{e}_t = G_0^{-1} \cdot \overrightarrow{u}_t$$

The parameters in this structural VAR were estimated from the US data. The estimated parameter values are the following:

$$\Gamma_1 = \begin{pmatrix} 0.907259 & -0.002406 \\ 0.809315 & 1.009731 \end{pmatrix} \tag{23}$$

$$\Lambda = \begin{pmatrix} 0.021903 \\ -0.123570 \end{pmatrix} \tag{24}$$

$$G_0^{-1} = \begin{pmatrix} 1 & 0 \\ 1.025198 & 1 \end{pmatrix} \tag{25}$$

and
$\sigma_{u_1} = 0.028816$
$\sigma_{u_2} = 0.078511$

The eigenvalues of the Γ_1 matrix are the following: 0.93246 and 0.98453. Since the absolute value of each eigenvalue is less than one, the stability of the VAR system above is guaranteed.

The implications of this calibration for the interaction between fiscal and monetary policies can be illustrated by the impulse response functions of the government's estimated structural VAR in Figure 3. The left column shows the effect of a positive fiscal shock (i.e., an unexpected increase in the fiscal primary deficit) on $\frac{F}{M}$ and $\frac{B}{M}$. As can be observed, the shock has a persistent effect on $\frac{F}{M}$ for a while, but eventually an offsetting surplus arises; so the effects of fiscal shocks are partially offset by future fiscal policy itself. On the other hand, the fiscal shock causes an increase in the $\frac{B}{M}$ ratio, but this ratio eventually returns to its original level because (i) future fiscal policy partially offsets the initial shock, and (ii) monetary policy accommodates in the long run increasing the money supply at a greater rate than the public bond supply.

The right column of Figure 3 shows the effect of a restrictive monetary policy shock (i.e., an unexpected increase in $\frac{B}{M}$). As expected, the $\frac{B}{M}$ ratio jumps up on impact and eventually returns to its original level. For this to happen, again two forces are at work: future open market operations that tend to offset the initial shock, and also an additional fiscal effort. Indeed, the $\frac{F}{M}$ ratio decreases below its original level for a while as a result of the monetary shock, so the fiscal policy also helps to preserve the intertemporal government's balance.

3.1.2 Solution

The effect of a monetary policy shock is presented in Figure 4. The behavior of the $\frac{B}{M}$ and $\frac{F}{M}$ ratios is, as it should be, what the calibration above implies. Again, the basic liquidity effect is present: the nominal interest rate jumps up on impact and then converges to its original level, following the behavior of the $\frac{B}{M}$ ratio. The inflation rate drops on impact, then it jumps up and converges to its original level from above. The demand for real money balances behaves as the mirror of the nominal interest rate, as expected.

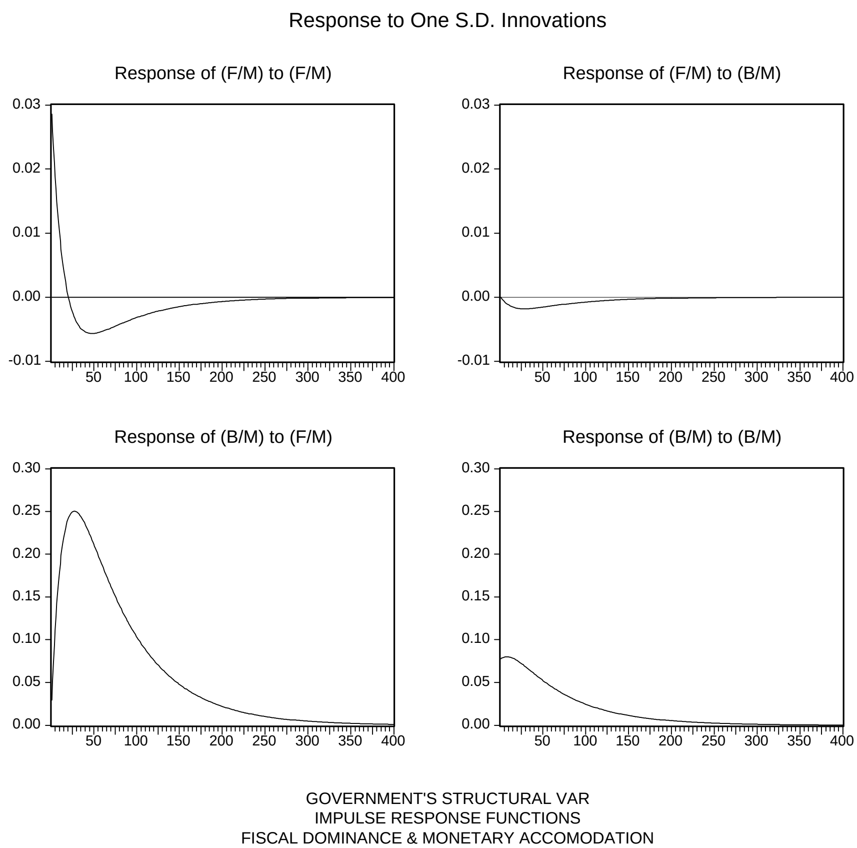

Figure 3: Fiscal Dominance and Monetary Accomodation: VAR Case

The nominal money supply drops on impact as a result of the unexpected open market operation, and then it increases at a rate greater than its steady state rate of growth. The same happens to the price level.

Comparing this model with the basic model above, we observe basically the same qualitative effects of a monetary policy shock in both cases. There are two exceptions. The first one, of course, is that in this VAR-policy case the $\frac{F}{M}$ ratio reacts to the monetary policy shock in a way such that fiscal policy helps to preserve the fiscal balance. The second exception is the behavior of the inflation rate. Even though in both cases the price level grows after the shock at a rate greater than its steady state rate of growth, in the basic model the price level is not affected at the moment of the shock (so the inflation rate in that period is equal to its steady state value), while in the VAR-policy case the inflation rate drops on impact. This result is due to the fact that the fiscal contribution in the VAR case makes it possible to preserve the government's intertemporal balance with less seigniorage revenue than in the basic case; hence, less inflation is expected and the demand for real money balances falls less in the VAR case than in the basic case. Consequently, the initial drop in the nominal money supply caused by the shock must be accompanied by an initial drop in the price level.

Anticipated Monetary Shock What happens if the monetary policy shock is announced in advance? Figure 5 illustrates this case. In particular, suppose that at the beginning of the current period (period 0) it is announced that at the beginning of period 4 there will be a monetary policy shock such that $u_{2,t+4} = \sigma_{u_2}$. No other policy shock (monetary or fiscal) is expected to occur. The policy variables $\frac{B}{M}$ and $\frac{F}{M}$ are expected to behave according to their joint (VAR) law of motion.

As we saw in Figure 4, when the shock is not announced before it occurs, the price level drops on impact, causing a sudden increase in the real value of the money balances. If the shock is announced in advance, because of a non-arbitrage argument, the demand for money balances increases immediately, requiring the price level to drop at the time of the announcement. Of course, the nominal interest rate must drop at the time of the announcement to be consistent with the increase in the demand for real money balances. So notice that the announcement of a policy that will cause an increase in the interest rate in the future (the liquidity effect), actually causes the nominal

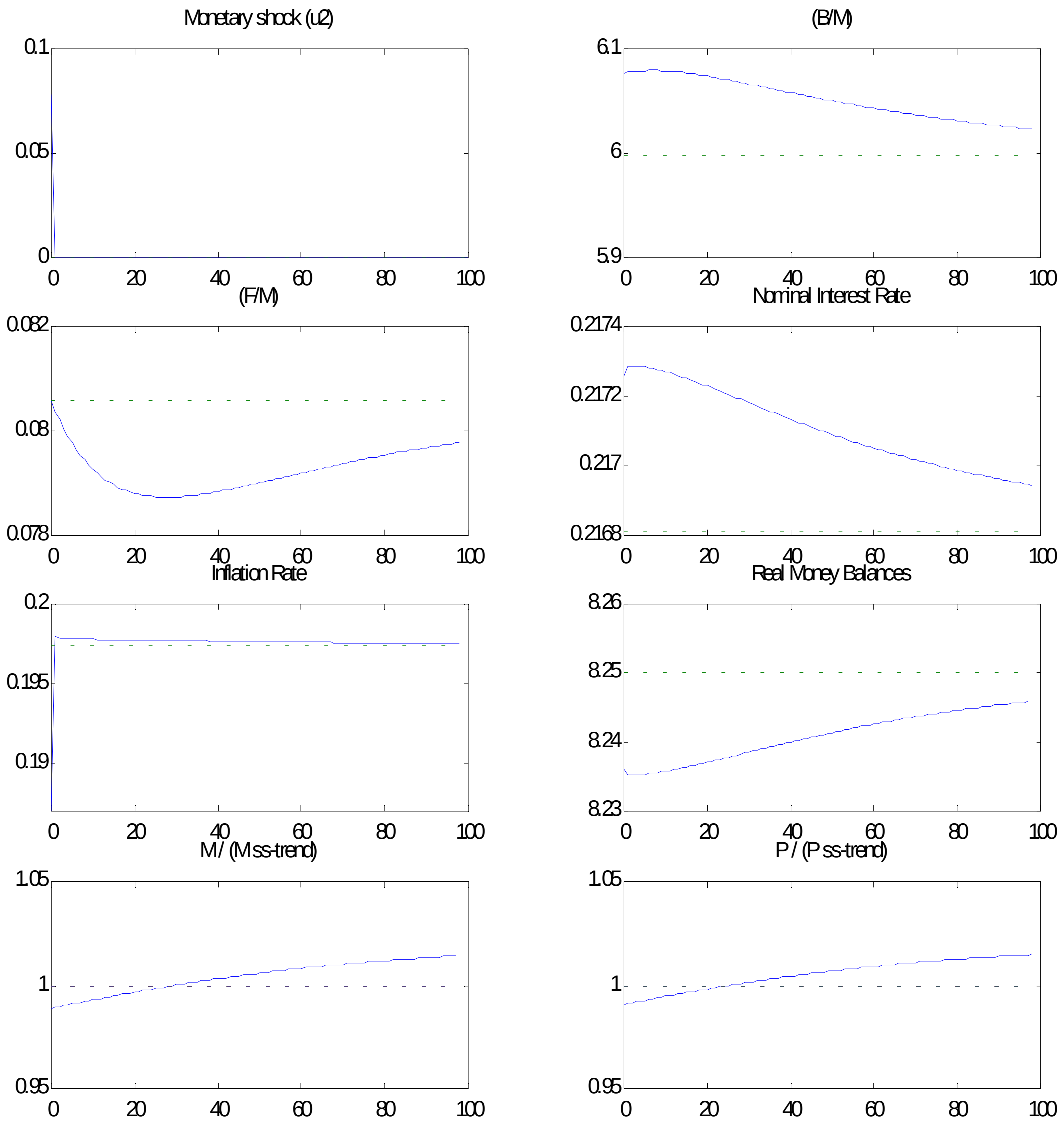

Figure 4: Monetary Policy Shock; Dominant Fiscal & Accommodating Monetary Policies; VAR Case

interest rate to drop at the moment of the announcement. The decrease in the nominal interest rate causes a decrease in the rate of growth of the money supply, since the government needs to issue less new money to meet the interest payments on the public debt. The inflation rate drops drastically at the time of the announcement; then its level increases but not enough to reach its steady state value.

At the time of the shock, the announced open market operation occurs. This implies that $\frac{B}{M}$ jumps up and the nominal money supply jumps down. The expected behavior of the $\frac{B}{M}$ and $\frac{F}{M}$ ratios implies that the nominal money supply will increase at a greater rate in the future. Hence, the expected inflation rate jumps up, and so does the nominal interest rate, materializing the liquidity effect. The demand for real money balances drops as the nominal interest rate jumps up. The price level begins to increase at a greater rate, but it does not jump drastically[15] when the shock occurs as it does when the shock is announced.

The trajectories for the nominal interest rate, the inflation rate, the demand for real money balances, the ratio of nominal money balances to its trend in steady state, and the ratio of the price level to its trend is steady state, between the announcement of the shock and the shock itself, are explosive. However, at the time of the shock all the variables engage in trajectories that converge to the steady state.

3.2 Monetary Dominance and Fiscal Accommodation

This institutional arrangement corresponds to the case in which the monetary authority exogenously determines the rate of growth of the money supply without being constrained by the government's revenue requirements. In order to preserve the government's intertemporal balance, the fiscal authority must alter the path of the fiscal primary deficit whenever the monetary authority alters the rate of growth of the money supply and, hence, the present value of the seigniorage revenue.

In this case we assume that monetary policy is characterized by a stationary stochastic process for the $\frac{M_{t+1}}{M_t}$ ratio, while fiscal policy is characterized

[15]In a continuous time model, the path for the price level would not display any discontinuity at the time of the shock, but it would at the time of the announcement. Of course, the path for the inflation rate would be discontinuous at both the time of the announcement and the time of the shock itself, as it is in the discrete time model.

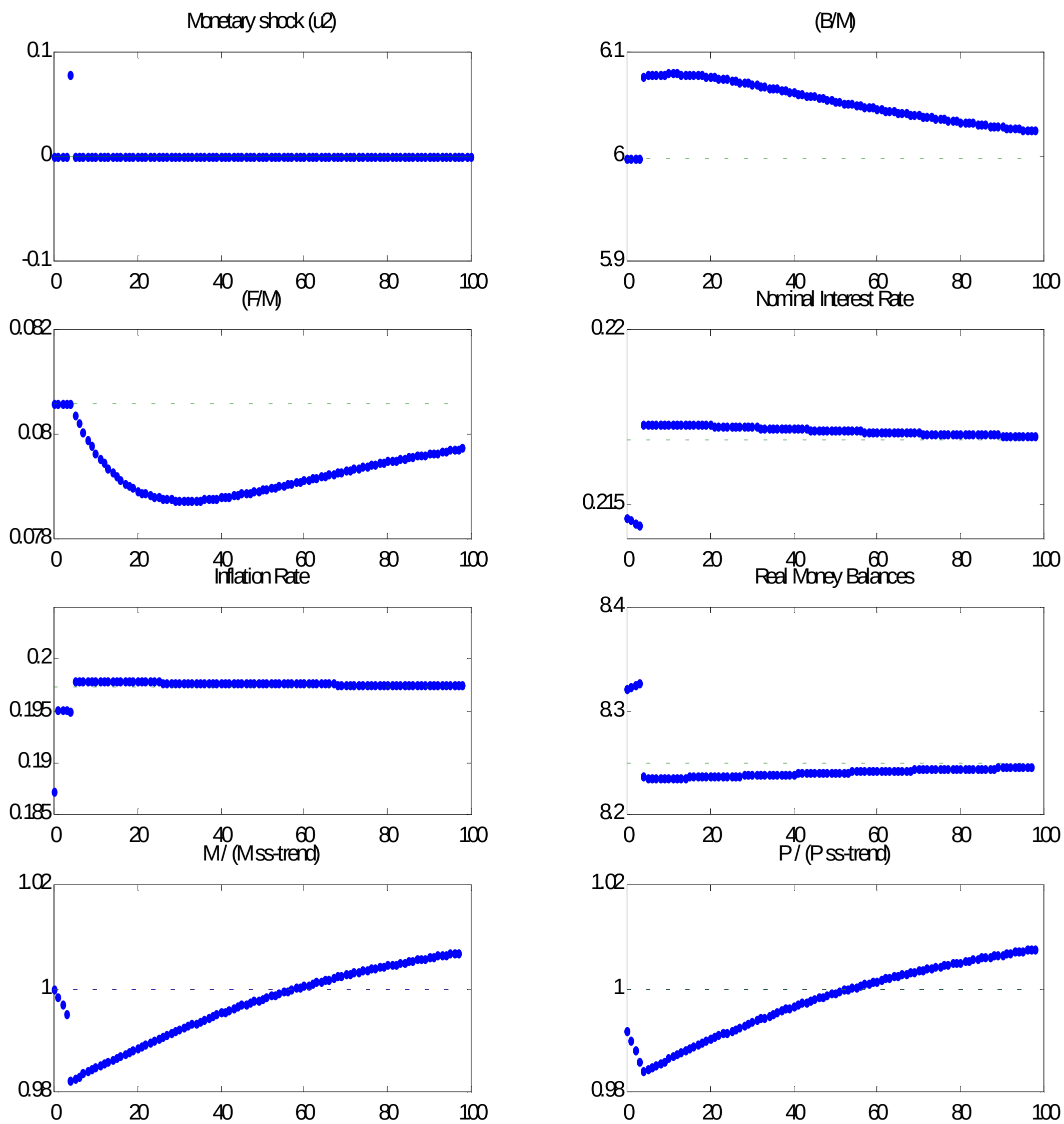

Figure 5: Fiscal Dominance and Monetary Accommodation: Monetary policy shock announced 4 periods in advance

by a stationary stochastic process for the $\frac{B}{M}$ ratio. The fiscal primary deficit must vary in a period by period basis to satisfy the government's period budget constraint. We now present the model for the government's behavior for this case in detail, using a general framework that will be applied in both a basic case and a VAR case (the distinction between these two cases will be clarified as we calibrate the model).

Let us define the following vectors:

$$\overrightarrow{g}_t = \left[\left(\frac{M_{t+1}}{M_t} \right), \left(\frac{B_{t+1}}{M_{t+1}} \right) \right]' \tag{26}$$

$$\overrightarrow{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{27}$$

where q_1 and q_2 are constants.

Now, we assume that the vector $\overrightarrow{g}_t$ obeys the following stationary, structural, recursive VAR process:

$$G_0 \cdot \overrightarrow{g}_t + G(Ł) \cdot \overrightarrow{g}_{t-1} = \overrightarrow{q} + \overrightarrow{u}_t \tag{28}$$

where G_0 is either a lower-triangular or a diagonal, 2×2 matrix with ones in its diagonal; $G(Ł)$ is a matrix polynomial in the lag operator Ł; and $\overrightarrow{u}_t$ is a vector stochastic process such that:

$$\overrightarrow{u}_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \sim iid, N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{u_1} & 0 \\ 0 & \sigma^2_{u_2} \end{bmatrix} \right) \tag{29}$$

The reduced form of (28) is the following:

$$\overrightarrow{g}_t = \Gamma(Ł) \cdot \overrightarrow{g}_{t-1} + \Lambda + \overrightarrow{e}_t \tag{30}$$

where

$$\Gamma(Ł) = -G_0^{-1} \cdot G(Ł) \tag{31}$$

$$\Lambda = G_0^{-1} \cdot \overrightarrow{q} \tag{32}$$

$$\overrightarrow{e}_t = G_0^{-1} \cdot \overrightarrow{u}_t \tag{33}$$

Now $\frac{M_{t+1}}{M_t}$ and $\frac{B}{M}$ determine together the path for the primary fiscal deficit. Dividing the period budget constraint over M_t and solving for $\frac{F}{M}$, we get:

$$\left(\frac{F_t}{M_t}\right) = \left(\frac{M_{t+1}}{M_t}\right) \cdot \left[1 + \left(\frac{B_{t+1}}{M_{t+1}}\right)\right] - \left\{(1+i_t)\left(\frac{B_t}{M_t}\right)\right\} - 1 \tag{34}$$

3.2.1 Basic Case

In the basic case we assume that $\frac{M_{t+1}}{M_t}$ and $\frac{B}{M}$ are independent, stationary stochastic processes. This implies that the rate of growth of the money supply is completely unaffected by current or future actions of the fiscal authority. Moreover, the monetary authority is not affected by the implications of its own actions on the government's intertemporal balance. For instance, if today's monetary shock decreases (ceteris paribus) the present value of seigniorage revenue, the monetary authority's future actions are not required to offset (either totally or partially) this effect. Hence, the burden of preserving the government's intertemporal balance is basically in the fiscal authority's hands, which must modify the primary fiscal deficit to accomplish this task.[16]

The government's model above is specialized as follows in order to characterize this simple case: G_0 and all matrices in the $G(\mathrm{L})$ matrix polynomial are diagonal matrices (in particular, G_0 is the identity matrix).

Calibration The stochastic processes for the government's policy variables were approximated by the following degenerated, reduced-form VAR process:

$$\overrightarrow{g}_t = \Gamma_1 \cdot \overrightarrow{g}_{t-1} + \Lambda + \overrightarrow{e}_t \tag{35}$$

[16]The fiscal effort can be helped by a change in the price level that modifies the real value of the government's nominal liabilities.

where:

$$\overrightarrow{e}_t = G_0^{-1} \cdot \overrightarrow{u}_t$$

The parameters corresponding to the stochastic process for the $\frac{M_{t+1}}{M_t}$ ratio were estimated from the US monetary base series for the period 1959-1997. Those corresponding to the stochastic process for the $\frac{B}{M}$ ratio were taken from Castañeda [2]. The estimated parameter values are the following[17]:

$$\Gamma_1 = \begin{pmatrix} 0.725699 & 0 \\ 0 & 0.998 \end{pmatrix} \tag{36}$$

$$\Lambda = \begin{pmatrix} 0.278782 \\ 0.008 \end{pmatrix} \tag{37}$$

$$G_0^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{38}$$

and
$\sigma_{u_1} = 0.004760$
$\sigma_{u_2} = 0.115$

Solution The effect of a monetary shock in this institutional setting is shown in Figure 6. The rate of growth of the money supply jumps up at the time of the shock and then returns asymptotically to its original level. The fiscal variable $\frac{B}{M}$ remains constant, since it is assumed to be independent from the monetary variable $\frac{M_{t+1}}{M_t}$. However, issuing more money increases the amount of resources available to the public sector. Since future seigniorage is not expected to decrease below its original level, preserving the government's intertemporal balance requires the fiscal primary deficit ($\frac{F}{M}$) to increase; and this is exactly what happens, confirming the accommodating role of fiscal policy.

The response of the nominal interest rate to the monetary policy shock is exactly the opposite to the liquidity effect: the interest rate jumps up on impact and then converges asymptotically to its original level. This result is

[17] The off-diagonal elements of Γ_1 are restricted to be zero, and the G_0^{-1} matrix is restricted to be equal to the identity matrix.

a straightforward consequence of the behavior of the expected inflation rate, which in turn mimics the rate of growth of the money supply. Of course, the nominal money supply jumps on impact and remains above its steady state trend. The price level jumps even more on impact, since the increase in the nominal money supply is accompanied by a decrease in the demand for real money balances explained, in turn, by the increase in the nominal interest rate.

The effect of a fiscal shock in this environment can be observed in Figure 7. Basically, there is no effect at all, since this is a textbook example of the Ricardian Equivalence result. The fiscal shock corresponds to a jump of the $\frac{B}{M}$ ratio; this ratio returns to asymptotically to its original level thereafter. Current and future values of the rate of growth of the money supply are completely unaffected by the fiscal shock (the 'monetary dominance' assumption). Hence, the increase in $\frac{B}{M}$ must be caused by an increase in the rate of growth of the nominal supply of bonds; the counterpart of that increase in the stock of public debt is an increase in the fiscal primary deficit $\frac{F}{M}$ at the moment of the shock.[18] The assumption that current and future seigniorage is unaffected by the shock implies that the offsetting effort to preserve the government's intertemporal balance must come from the fiscal policy. That is why the fiscal primary deficit remains below its original level for a long period while the $\frac{B}{M}$ ratio returns to its steady state value.

Since the path for the nominal money supply does not change after the fiscal shock, neither do the inflation rate, the nominal interest rate, and the demand for real money balances.

3.2.2 VAR Case

Now we assume that each policy variable reacts to the lagged values of the other policy variable (as well as to its own lagged values). In principle, this is a relaxed case of monetary dominance and fiscal accommodation because the burden of returning the $\frac{B}{M}$ ratio to its original level after a shock occurs

[18]It would probably seem more natural and intuitive to think of the fiscal shock as an increase in the primary fiscal deficit (because of a war or a natural disaster, for instance), rather than as an increase in $\frac{B}{M}$. However, since the stationarity of the $\frac{B}{M}$ process is crucial for the results of the model, we prefer to identify fiscal policy as the stationary, stochastic process for $\frac{B}{M}$ and fiscal shocks as shocks to this ratio.

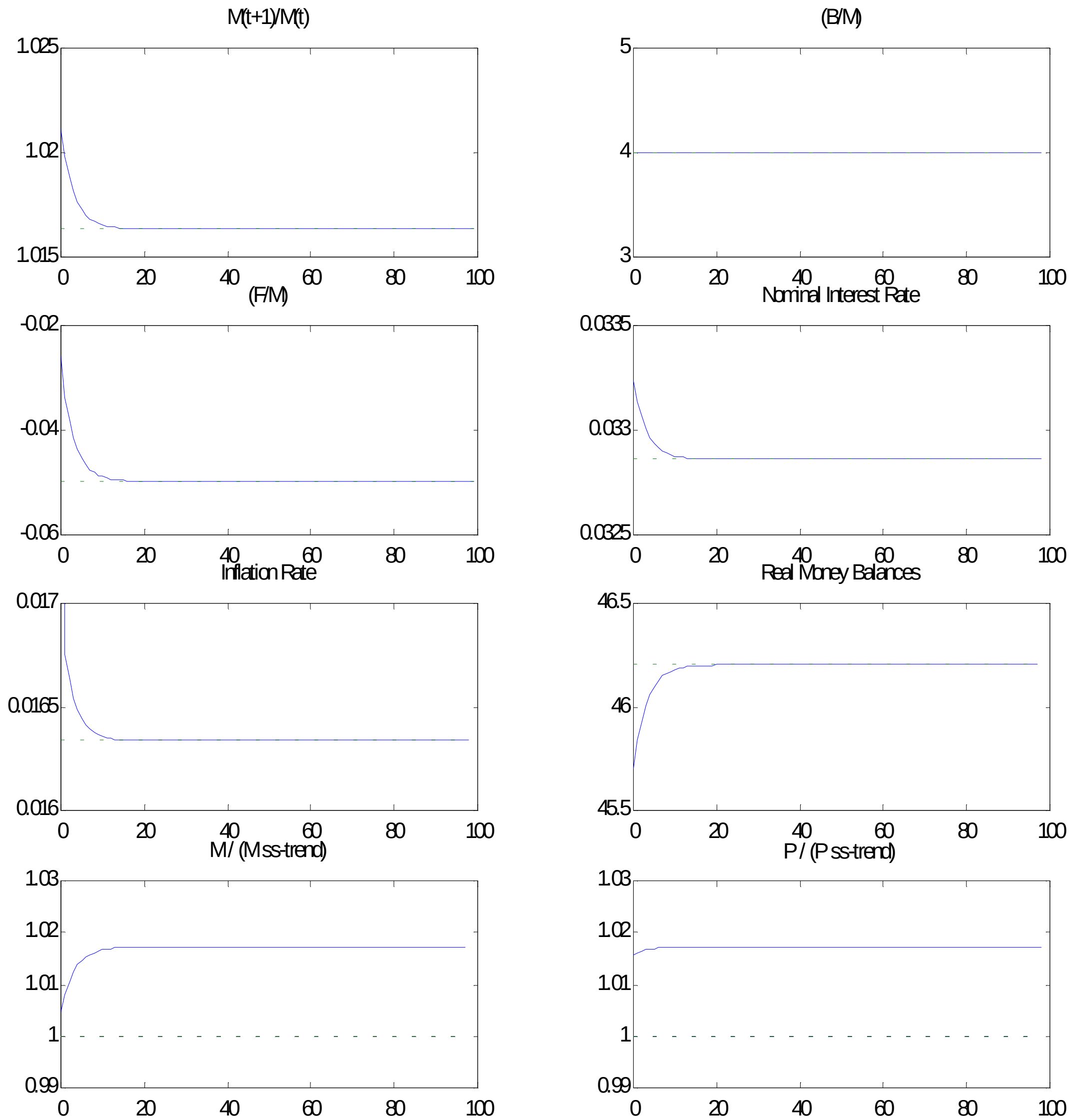

Figure 6: A Monetary Policy Shock (dominant monetary policy and accommodating fiscal policy)

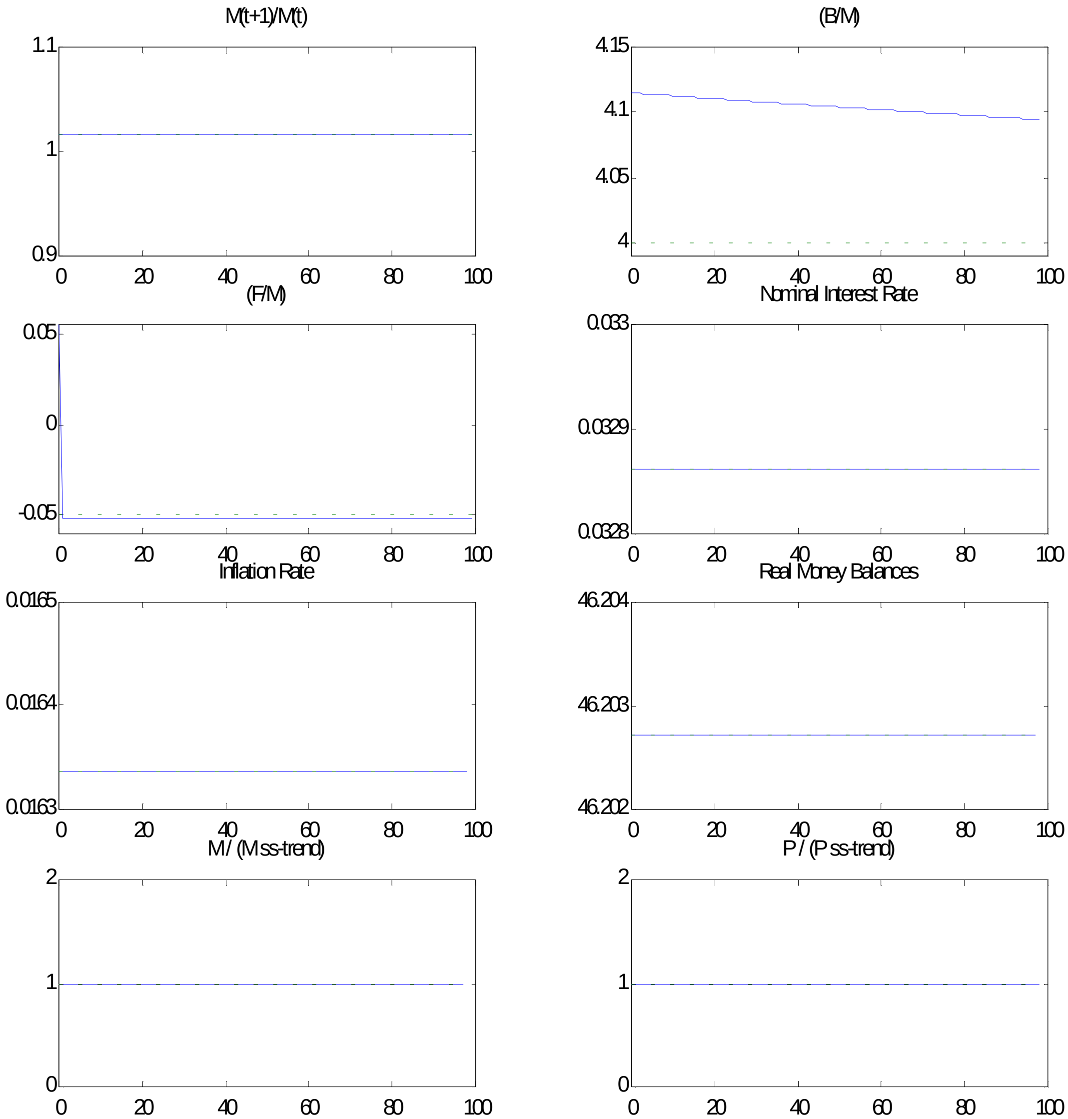

Figure 7: A Fiscal Shock: dominant monetary policy and accommodating fiscal policy

is shared by the two authorities (the particular way in which that happens depends on the VAR parameters).

Calibration The parameters for this case were directly estimated from the US data, for the period 1982-1999. The estimated parameter values are the following[19]:

$$\Gamma_1 = \begin{pmatrix} 0.678784 & -0.000350 \\ -0.575035 & 0.949624 \end{pmatrix} \tag{39}$$

$$\Lambda = \begin{pmatrix} 0.329590 \\ 0.988401 \end{pmatrix} \tag{40}$$

$$G_0^{-1} = \begin{pmatrix} 1 & 0 \\ -0.702415 & 1 \end{pmatrix} \tag{41}$$

and
$\sigma_{u_1} = 0.004668$
$\sigma_{u_2} = 0.098088$

The eigenvalues of the Γ_1 matrix are both less than one in absolute value (0.67804 and 0.95037), so the VAR system is stable.

The implications of this calibration for the interaction between fiscal and monetary policies can be illustrated by the impulse response functions of the government's estimated structural VAR in Figure 8. The left column shows the effect of a positive monetary shock (an unexpected increase in the $\frac{M_{t+1}}{M_t}$ ratio). The rate of money growth decreases monotonically to its original level, while the $\frac{B}{M}$ ratio drops on impact and then returns to its original level from below. The interpretation is that monetary policy is increasing the present value of seigniorage revenue. At the beginning, this extra revenue is used to redeem public debt, causing $\frac{B}{M}$ to drop; but in order to preserve the government's intertemporal balance, the fiscal primary deficit increases above its steady state level, causing $\frac{B}{M}$ to return to its unconditional mean.

The right column shows the effect of a fiscal shock (an unexpected increase in the $\frac{B}{M}$ ratio). While the $\frac{B}{M}$ ratio jumps up and returns asymptotically to its original level, the rate of growth of money slightly decreases for

[19] G_0^{-1} is restricted to be lower diagonal, with ones on its diagonal.

a while. In this case, the monetary authority does not help the fiscal authority to preserve the government's intertemporal balance; on the contrary, it 'punishes' the fiscal authority by slightly decreasing the present value of seigniorage revenue, so future fiscal deficits need to be lower (while $\frac{B}{M}$ has not converged) to offset not only the original fiscal shock but also the monetary authority's reaction.

Solution Figure 9 presents the effects of an expansive monetary shock in this environment. The results are basically the same that were obtained in the simple case of monetary dominance, so the interaction between monetary and fiscal policies that is allowed by the VAR specification does not seem to be too important. In Figure 9 we can see that, also in this case, the opposite of a liquidity effect obtains. In particular, the nominal interest rate jumps up on impact and converges asymptotically to its original level, and so does the inflation rate. This is the result of the temporary increase in the rate of growth of the money supply that is announced by the shock. The fiscal deficit needs to increase temporarily to offset the effect of the additional seigniorage revenue on the government's intertemporal balance.

The effect of a fiscal shock in this case (Figure 10) is somewhat different from the simple case. In the simple case, a typical Ricardian Equivalence result obtained, in which only the path of the $\frac{B}{M}$ ratio and that of the fiscal primary deficit were affected and all other variables were unaffected. In the VAR case, the monetary policy variable $\frac{M_{t+1}}{M_t}$ reacts (decreases slightly and temporarily) to the fiscal shock. Hence, the present value of seigniorage revenue diminishes and less inflation is expected in the future. The nominal interest rate, consequently, drops and remains below its steady state level temporarily. The nominal money stock and the price level remain permanently below their levels in the absence of any shock.

4 CONCLUSION

This paper presents and simulates a simple model that produces a liquidity effect in a dynamic, general equilibrium macroeconomic model with flexible prices and without limited participation constraints. The basic result is that the liquidity effect can be explained by the expected inflation effect of

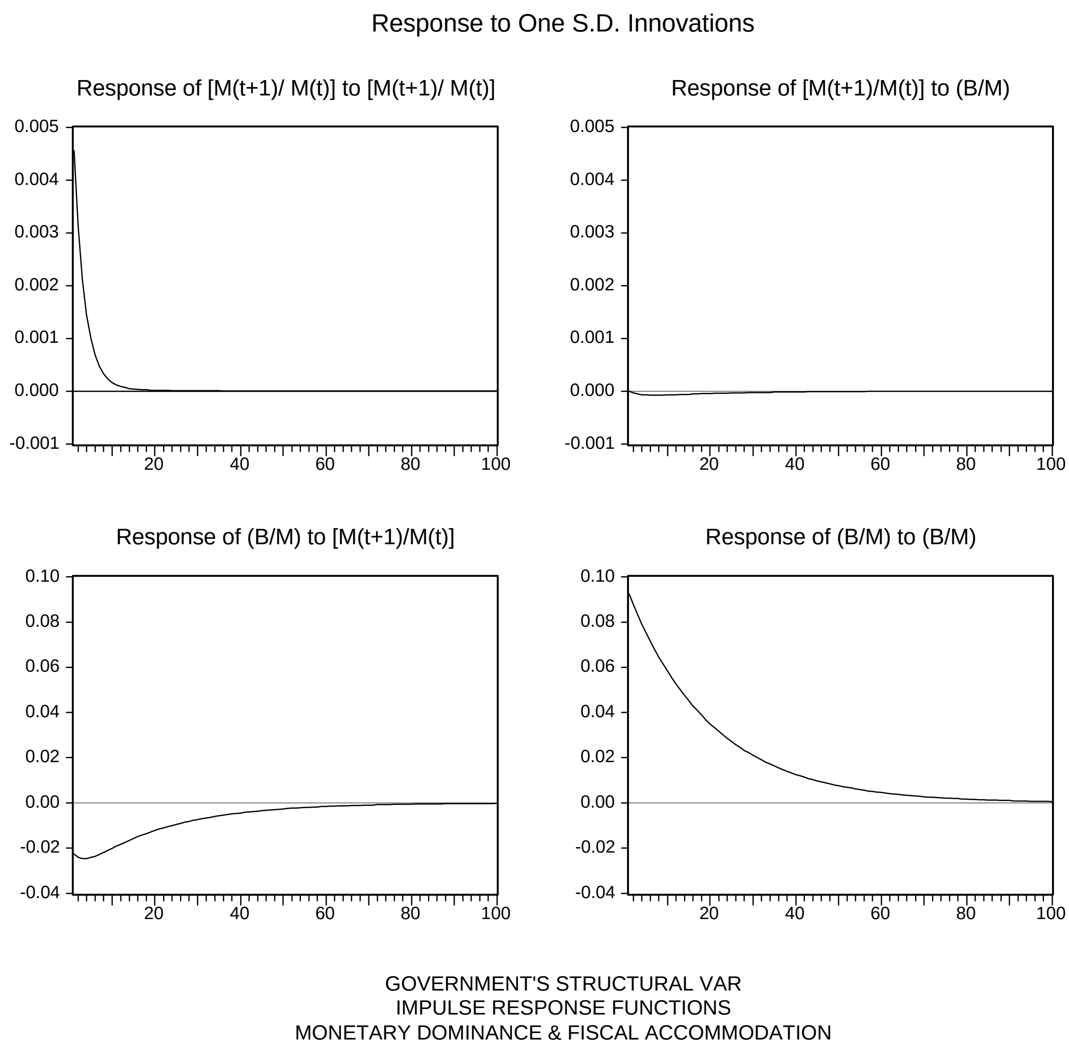

Figure 8: Government's Structural VAR Impulse-Responses: Monetary Dominance and Fiscal Accommodation

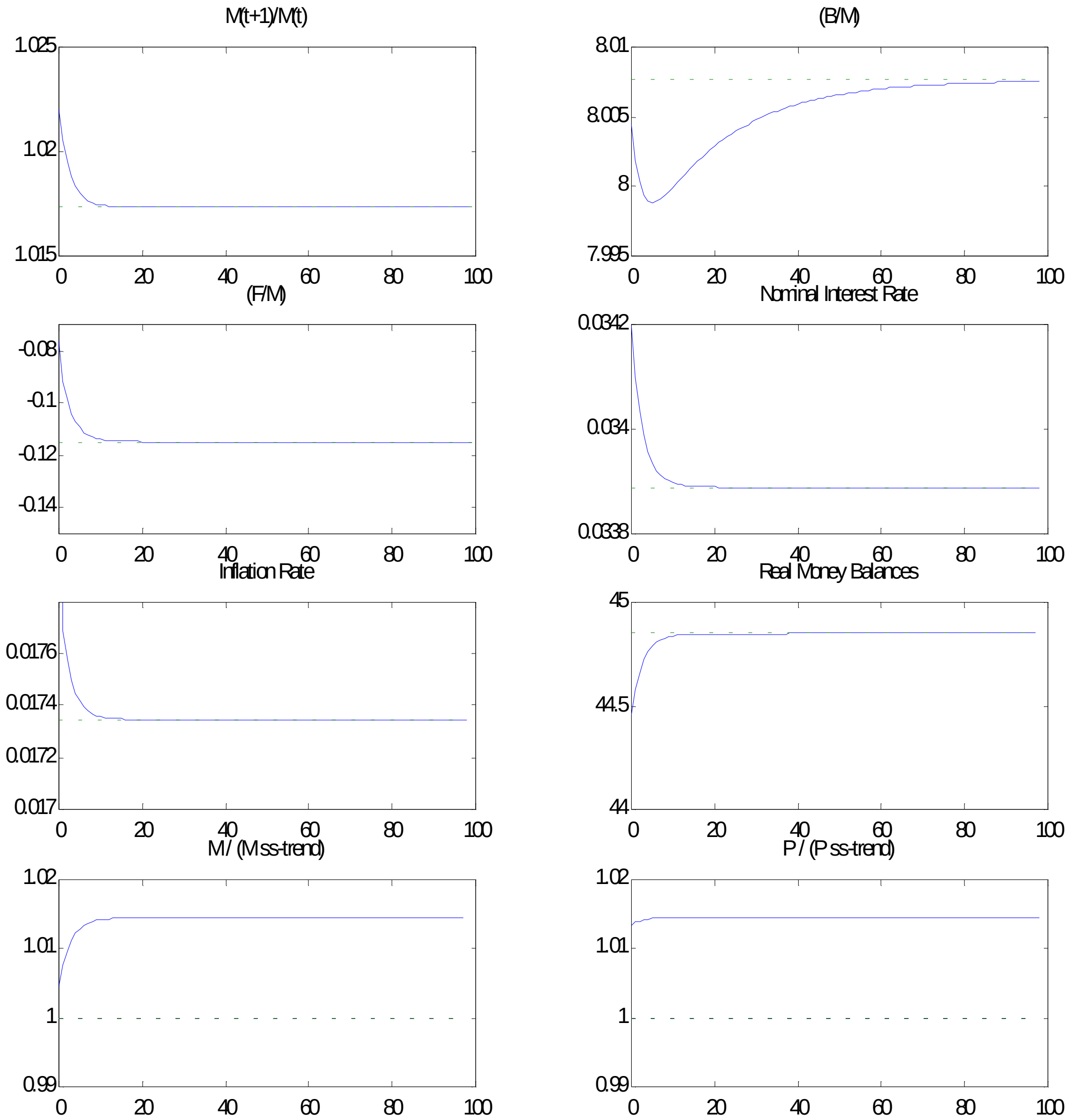

Figure 9: Monetary Policy Shock: Monetary Dominance and Fiscal Accommodation with structural VAR for policy variables

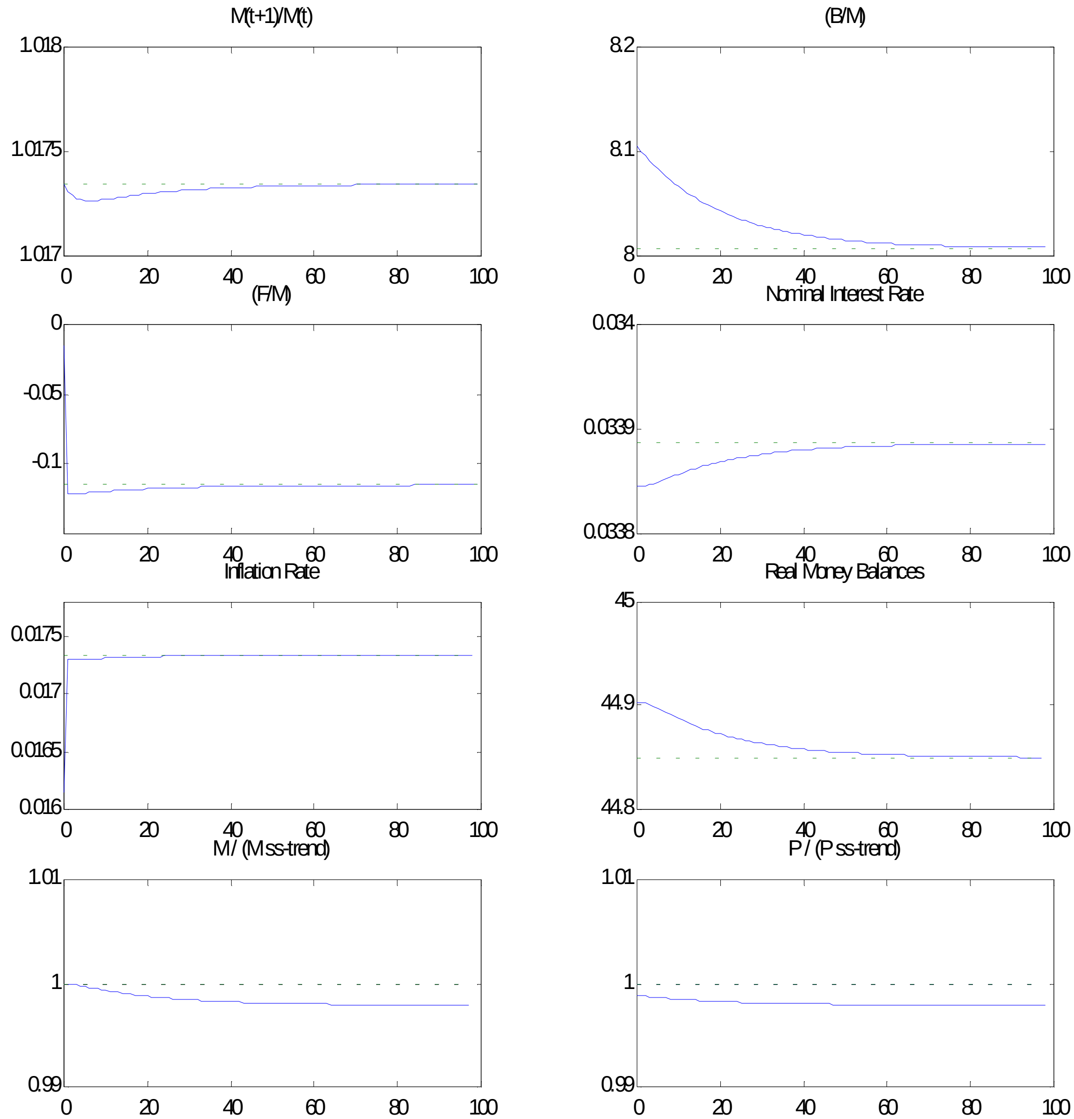

Figure 10: Fiscal Policy Shock: Monetary Dominance and Fiscal Accommodation with structural VAR for policy variables

a monetary policy shock (i.e., an unexpected open market operation) that announces open market operation of opposite sign to the original shock.

This mechanism works in an institutional arrangement in which the fiscal authority decides the path of the fiscal primary deficit without regard of the government's intertemporal budget constraint and the monetary authority is required to adjust the flow of seigniorage revenue in the long run in order to preserve the government's balance. In particular, fiscal policy is modeled as a stationary stochastic process for the ratio of the fiscal primary deficit to the money supply (which in this simple model equals the monetary base); and monetary policy is modeled as a stationary, stochastic process for the ratio of the nominal public debt to the money supply. This specification for the monetary policy implies that the monetary authority is eventually forced to partially monetize the deficits in order to prevent the debt-to-money ratio from exploding. The simulations were based on plausible calibrations of the stochastic processes to match some features of the US historical data.

Alternative institutional arrangements were also explored. It was showed that relaxing the assumption of independence between the fiscal and the monetary stochastic processes still preserves the liquidity effect feature of the model, provided that the monetary authority is ultimately in charge of preserving the government's intertemporal balance. On the contrary, for an institutional arrangement of monetary dominance and fiscal accommodation, monetary policy shocks cause exactly the opposite of a liquidity effect. This result does not change if the independence between the fiscal and monetary stochastic processes is relaxed.

The relevance of the results obtained in this paper for monetary theory and policy depends on the nature of the institutional arrangements that coordinate the operation of fiscal and monetary policies in the long run. If for a particular country and historical period it is reasonable to think that the prevalent policy mix resembles a setup of fiscal dominance and monetary accommodation, then the results presented here might be a useful guide to identifying the effects of monetary policy shocks on the interest rate.

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