

A FCVAR MODEL FOR THE CENTRAL AMERICAN ECONOMY

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We use univariate and multivariate **I(d) techniques** in the analysis of the Central American economy by looking at three variables, **prices, interest rates and monetary base** on the six countries that form the **CMCA** (Costa Rica, Honduras, El Salvador, Guatemala, Nicaragua and Dominican Rep.)

Structure of the paper:

1. Introduction
2. Contextual setting
3. Methodology
4. Data and Empirical results
5. Concluding comments

1. INTRODUCTION

Traditionally, studies conducted by CMCA have focused on studying inflation levels and other variables using econometric methods such as unit roots, VAR, cointegration and VECM techniques.

In this article, we propose a new approach based on fractional integration and FCVAR models for the analysis of the Central American economy.

2. CONTEXTUAL SETTING

The CMCA attempts to provide economic and financial stability to five Central American countries (Costa Rica, El Salvador, Guatemala, Honduras and Nicaragua) and the Dominican Republic.

Between 1951 and 1957 several bilateral agreements were signed, constituting the basis for the creation a new system of Central Banks in Central America

July, 1961: Central American Compensation Chamber Agreement,

February 1964: Central American Monetary Union Establishment.

October 1974: Central American Monetary Agreement

December 1991: Tegucigalpa Protocol

October 1993: Guatemala Protocol

Despite not having as ultimate goal the adoption of a new common currency, the CMCA attempts to achieve economic and financial stability in the region, in order to promote the integration and mutual collaboration of its member countries.

3. METHODOLOGY

We use techniques based on the concept of

FRACTIONAL INTEGRATION

and its generalization to the multivariate case throughout the concept of

FRACTIONAL COINTEGRATION

Stationarity is a crucial concept to make statistical inference.

However, many macroeconomic time series are nonstationary.

Traditionally, two approaches to remove the nonstationarity:

1. Deterministic regressions on time
2. To take **FIRST DIFFERENCES**
(Unit roots)

FRACTIONAL INTEGRATION

It is a generalization of the concept of a unit root or first differences to the fractional case.

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots,$$

where u_t is $I(0)$ and d can be any real value.

$L =$ lag operator ($L^k x_t = x_{t-k}$)

Note that for all real value d ,

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \dots$$

such that $(1-L)^d x_t = u_t$ can be expressed as

$$x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \dots = u_t.$$

or

$$x_t = d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \dots + u_t.$$

In this context, the **fractional differencing parameter d** becomes the crucial parameter to indicate the degree of dependence or **persistence** in the data.

Thus, the higher the value of d is, the higher the degree of association between observations far away in time is.

Also, these processes admit a **MA(∞) representation**, such that

$$x_t = u_t + \gamma_1 u_{t-1} + \gamma_2 u_{t-2} + \gamma_3 u_{t-3} + \dots$$

and $\gamma_j = \Gamma(j+d) / [\Gamma(j+1)\Gamma(d)]$,

Γ = Gamma function,

and these coefficients (γ_s) depending on d , and decaying hyperbolically slowly to zero.

The $I(d, d > 0)$ processes are characterized because the density function $f(\lambda)$ is unbounded at the origin, i.e., $f(0) \rightarrow \infty$, since

$$f(\lambda) \sim c_1 \lambda^{-2d} \quad \text{as } \lambda \rightarrow 0^+ \quad |c_1| < \infty$$

Also, the autocovariance function presents a hyperbolic decay such that:

$$\gamma_j \sim c_2 j^{2d-1} \quad \text{as } j \rightarrow \infty \quad |c_2| < \infty$$

Adelman (1965), Granger (1966).

The I(d) processes were introduced by

Granger (1980, 1981)

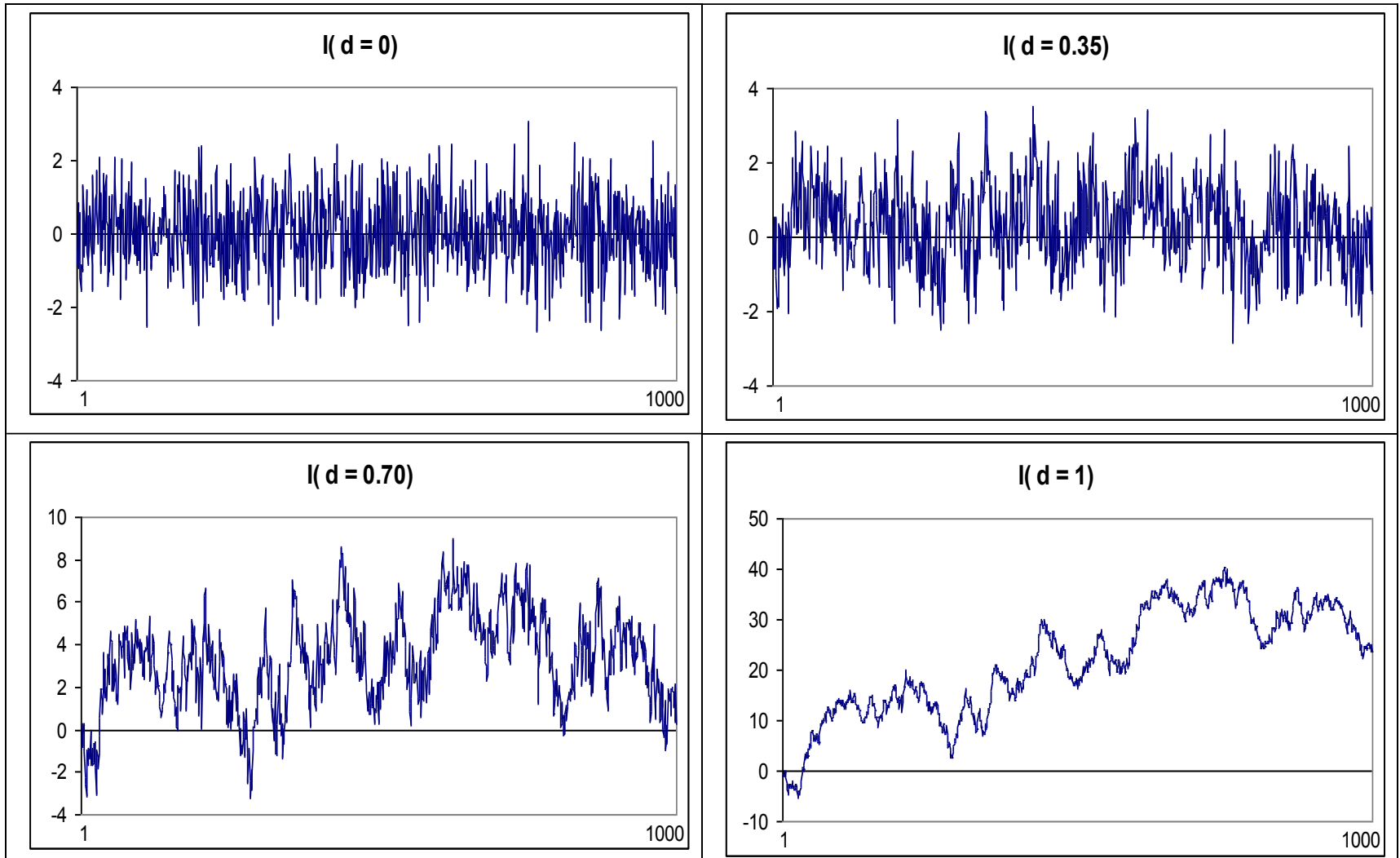
Granger and Joyeux (1980)

Hosking (1981)

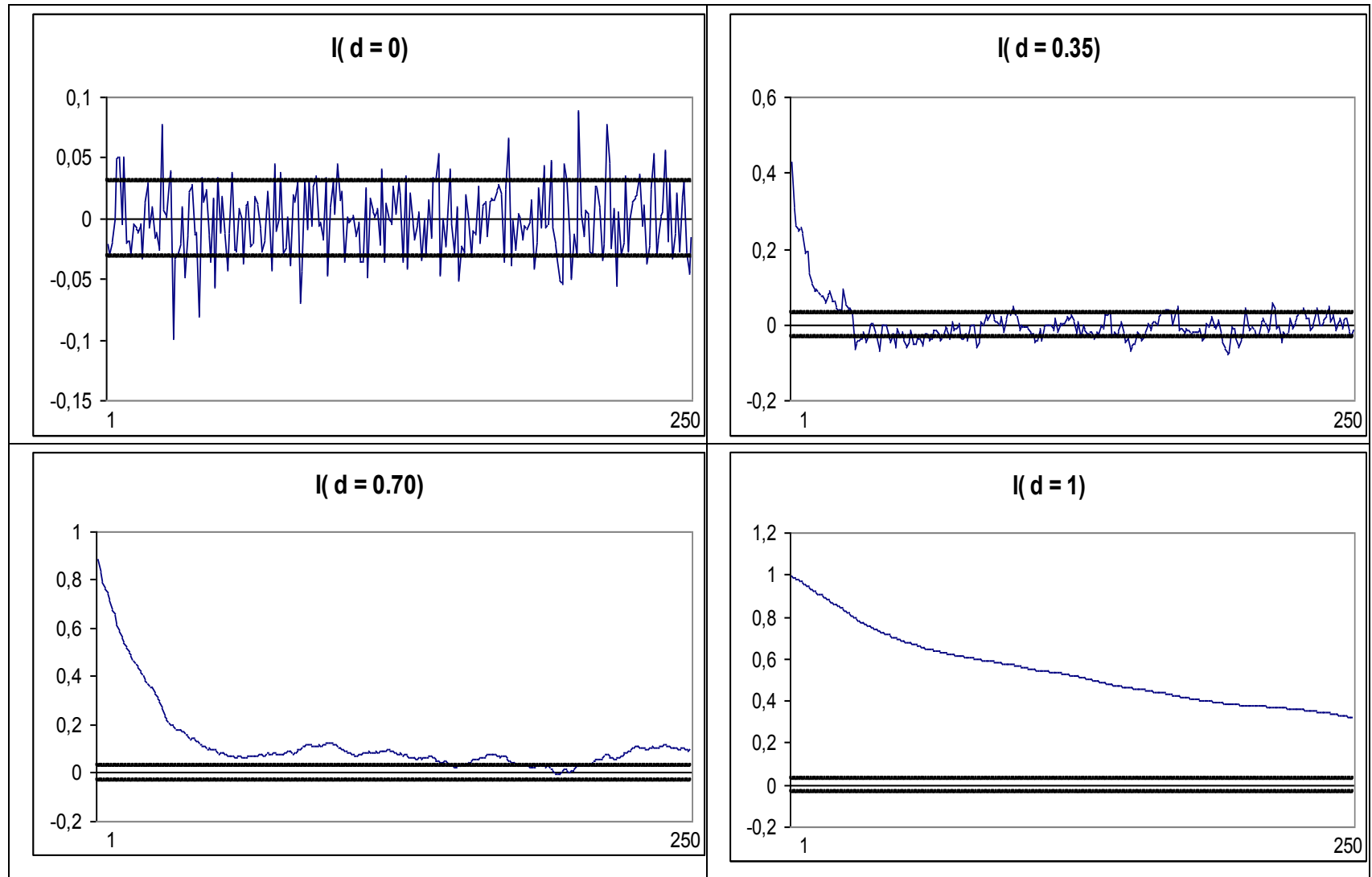
and they were justified in terms of the aggregation of heterogeneous processes by:

Robinson (1978), Granger (1980)

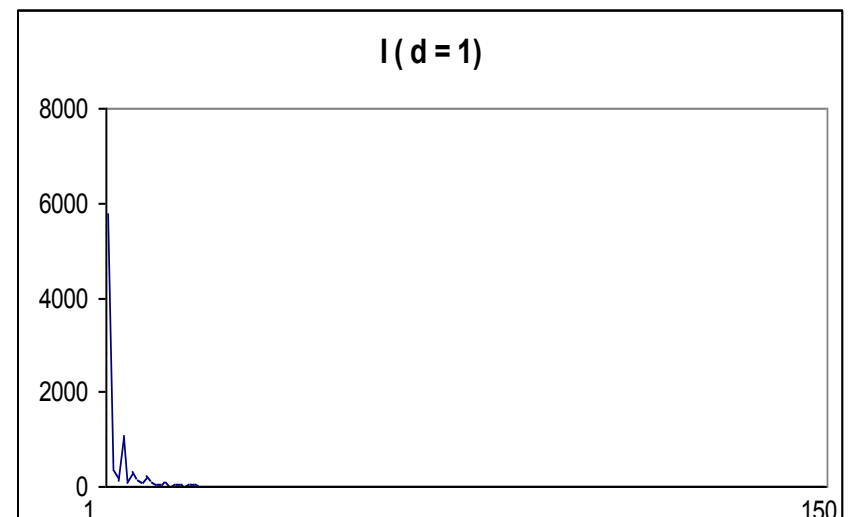
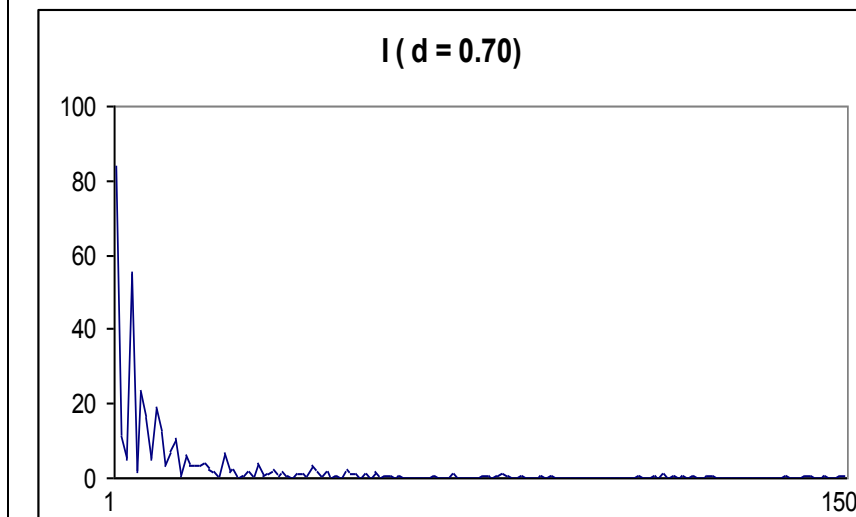
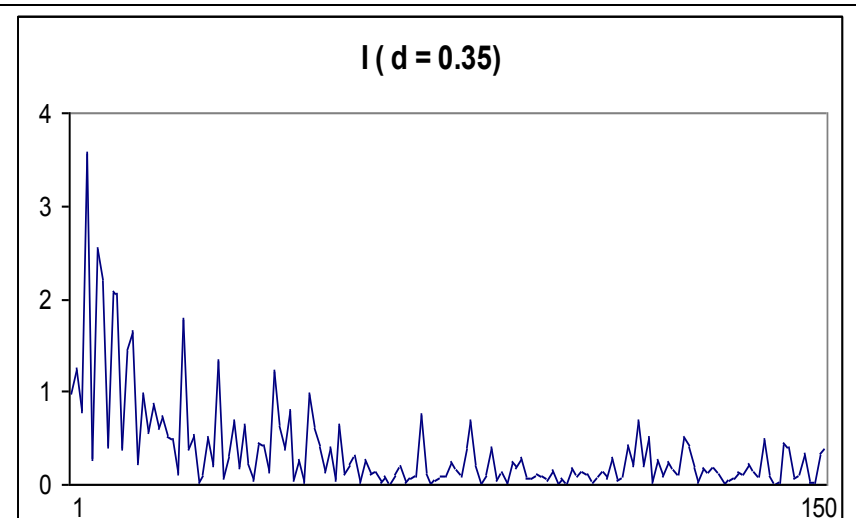
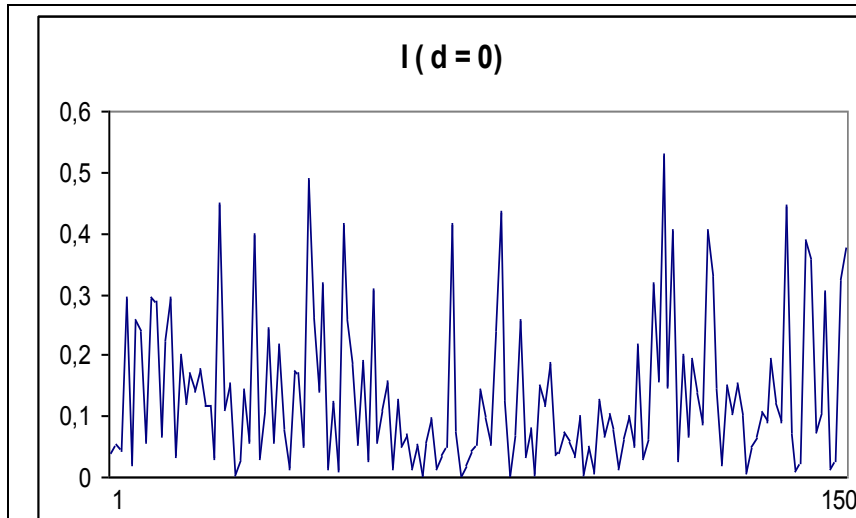
Simulations of $I(d)$ processes:



Correlograms of the simulated processes:



Periodograms of the simulated processes:



$$y_t \sim I(d)$$

- If $d = 0$, x_t is said to be short memory
- If $d > 0$, x_t is said to be long memory

- If $d < 0.5$, x_t is covariance stationary
- If $d \geq 0.5$, x_t is nonstationary

- If $d < 1$, x_t is mean reverting
- If $d \geq 1$, x_t is not mean reverting

The I(d) models have been widely employed to describe the behaviour of economic and financial time series data,

Diebold and Rudebusch (JME, 1989)

Sowell (JME, 1992a)

Cheung (JBES, 1993)

Baillie and Bollerslev (JOF, 1994)

Baillie (JOE, 1996)

Gil-Alana and Robinson (JOE, 1997)

The natural extension of fractional integration is the concept of:

FRACTIONAL COINTEGRATION

Standard bivariate cointegration assumes integer degrees of differentiation:

$$X_{1t} \sim I(1) \quad \text{and} \quad X_{2t} \sim I(1)$$

$$X_{1t} - \beta X_{2t} \sim I(0).$$

Engle and Granger (1987), Johansen (1996).

The extension to the bivariate fractional case:

$$X_{1t} \sim I(d) \quad \text{and} \quad X_{2t} \sim I(d)$$

$$X_{1t} - \beta X_{2t} \sim I(b).$$

with $b < d$, and where both d and b can be fractional values.

Cheung and Lai (1993), Gil-Alana (2003)

Johansen and Nielsen FCVAR (2012)

An expansion of the traditional and popular Johansen (96) CVAR approach.

It introduces fractional integration to the cointegrating framework.

Codes and instructions available on M. Nielsen's personal website.

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In most of its applications, the individual series are $I(d)$,

and the cointegrating relations are $I(0)$.

Dolatabadi, Nielsen and Xu (2014)

Nielsen, Jones and Popiel (2014)

Nielsen and Shibaev (2015)

Dolatabadi et al. (2015)

Nielsen and Popiel (2016)

4. EMPIRICAL RESULTS

DATA:

Data from January 2001 to December 2016
monthly, of

CPI

Monetary base, and

Interest rates

for the six countries that form the CMCA.

- We first perform **standard unit root methods** in the three series and the results support this hypothesis in the majority of the cases.
- However, unit root tests have extremely low power if the true DGP are fractionally integrated $I(d)$, with $d \neq 1$.
(Lee and Schmidt, 1996; etc.)

Then, we examine the model:

$$y_t = \beta_0 + \beta_1 t + x_t, \quad t = 1, 2, \dots$$

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots$$

assuming that the error term u_t is:

a) Uncorrelated (white noise)

B) Autocorrelated (Bloomfield)

A version of Robinson's (1994) LM tests.

Table 1: Estimates of d and 95% intervals under white noise disturbances

| PRICES | | | |
|-----------------------|-------------------|-------------------|---------------------|
| | No regressors | An intercept | A linear time trend |
| COSTA RICA | 0.98 (0.88, 1.10) | 1.41 (1.33, 1.51) | 1.35 (1.28, 1.48) |
| HONDURAS | 0.98 (0.89, 1.10) | 1.36 (1.26, 1.49) | 1.29 (1.20, 1.43) |
| EL SALVADOR | 0.98 (0.89, 1.10) | 1.15 (1.06, 1.28) | 1.14 (1.05, 1.26) |
| GUATEMALA | 0.98 (0.89, 1.10) | 1.38 (1.28, 1.52) | 1.34 (1.24, 1.48) |
| NICARAGUA | 0.98 (0.88, 1.10) | 1.38 (1.28, 1.53) | 1.37 (1.26, 1.52) |
| DOMINICAN | 0.98 (0.89, 1.10) | 1.51 (1.40, 1.65) | 1.49 (1.38, 1.63) |
| MONEY | | | |
| | No regressors | An intercept | A linear time trend |
| COSTA RICA | 0.97 (0.88, 1.09) | 1.02 (0.93, 1.12) | 1.01 (0.94, 1.10) |
| HONDURAS | 0.99 (0.90, 1.11) | 0.91 (0.79, 1.06) | 0.93 (0.83, 1.06) |
| EL SALVADOR | 0.97 (0.88, 1.09) | 0.77 (0.72, 0.84) | 0.76 (0.70, 0.84) |
| GUATEMALA | 0.98 (0.89, 1.11) | 0.75 (0.70, 0.85) | 0.71 (0.60, 0.86) |
| NICARAGUA | 0.98 (0.89, 1.10) | 0.88 (0.83, 0.95) | 0.86 (0.78, 0.93) |
| DOMINICAN | 0.98 (0.89, 1.09) | 1.01 (0.92, 1.13) | 1.01 (0.93, 1.12) |
| INTEREST RATES | | | |
| | No regressors | An intercept | A linear time trend |
| COSTA RICA | 0.99 (0.90, 1.11) | 1.35 (1.24, 1.48) | 1.35 (1.24, 1.47) |
| HONDURAS | 0.97 (0.88, 1.10) | 1.25 (1.17, 1.35) | 1.25 (1.17, 1.35) |
| EL SALVADOR | 0.95 (0.86, 1.06) | 0.91 (0.84, 0.99) | 0.91 (0.84, 0.99) |
| GUATEMALA | 0.97 (0.88, 1.09) | 1.13 (1.08, 1.20) | 1.12 (1.07, 1.18) |
| NICARAGUA | 0.92 (0.83, 1.04) | 0.52 (0.44, 0.61) | 0.54 (0.47, 0.63) |
| DOMINICAN | 1.01 (0.93, 1.12) | 1.13 (1.03, 1.25) | 1.13 (1.03, 1.25) |

Table 2: Estimates of d and 95% intervals under autocorrelated disturbances

| PRICES | | | |
|-----------------------|-------------------|-------------------|---------------------|
| | No regressors | An intercept | A linear time trend |
| COSTA RICA | 0.94 (0.80, 1.14) | 1.38 (1.26, 1.51) | 1.30 (1.21, 1.43) |
| HONDURAS | 0.95 (0.79, 1.15) | 1.23 (1.04, 1.43) | 1.14 (1.02, 1.32) |
| EL SALVADOR | 0.95 (0.79, 1.15) | 1.08 (0.95, 1.27) | 1.08 (0.96, 1.25) |
| GUATEMALA | 0.96 (0.79, 1.16) | 1.29 (1.10, 1.52) | 1.21 (1.07, 1.44) |
| NICARAGUA | 0.94 (0.80, 1.14) | 1.19 (1.03, 1.37) | 1.16 (1.03, 1.35) |
| DOMINICAN | 0.96 (0.82, 1.15) | 1.31 (1.16, 1.51) | 1.27 (1.13, 1.46) |
| MONEY | | | |
| | No regressors | An intercept | A linear time trend |
| COSTA RICA | 0.96 (0.80, 1.15) | 1.13 (0.98, 1.31) | 1.10 (0.99, 1.27) |
| HONDURAS | 0.96 (0.79, 1.16) | 0.85 (0.69, 1.34) | 0.91 (0.71, 1.26) |
| EL SALVADOR | 0.94 (0.80, 1.15) | 1.01 (0.87, 1.18) | 1.01 (0.86, 1.18) |
| GUATEMALA | 0.93 (0.79, 1.15) | 0.76 (0.70, 0.88) | 0.60 (0.45, 0.86) |
| NICARAGUA | 0.95 (0.79, 1.15) | 0.98 (0.90, 1.12) | 0.97 (0.86, 1.14) |
| DOMINICAN | 0.96 (0.80, 1.15) | 1.00 (0.84, 1.18) | 1.00 (0.89, 1.17) |
| INTEREST RATES | | | |
| | No regressors | An intercept | A linear time trend |
| COSTA RICA | 0.97 (0.81, 1.17) | 1.30 (1.07, 1.61) | 1.30 (1.07, 1.62) |
| HONDURAS | 0.93 (0.79, 1.15) | 1.39 (1.23, 1.61) | 1.38 (1.23, 1.60) |
| EL SALVADOR | 0.97 (0.81, 1.16) | 1.11 (0.98, 1.30) | 1.12 (0.98, 1.31) |
| GUATEMALA | 0.93 (0.79, 1.15) | 1.54 (1.40, 1.74) | 1.47 (1.35, 1.67) |
| NICARAGUA | 0.92 (0.77, 1.14) | 0.64 (0.47, 0.84) | 0.69 (0.56, 0.85) |
| DOMINICAN | 1.00 (0.85, 1.19) | 1.04 (0.85, 1.28) | 1.04 (0.85, 1.28) |

Table 3: Summary of the parametric results

| i) No autocorrelation | | | |
|------------------------------|-------------------|-------------------|-------------------|
| | PRICES | MONEY | INTEREST |
| COSTA RICA | 1.35 (1.28, 1.48) | 1.01 (0.94, 1.10) | 1.35 (1.24, 1.48) |
| HONDURAS | 1.29 (1.20, 1.43) | 0.93 (0.83, 1.06) | 1.25 (1.17, 1.35) |
| EL SALVADOR | 1.14 (1.05, 1.26) | 0.76 (0.70, 0.84) | 0.91 (0.84, 0.99) |
| GUATEMALA | 1.34 (1.24, 1.48) | 0.71 (0.60, 0.86) | 1.12 (1.07, 1.18) |
| NICARAGUA | 1.37 (1.26, 1.52) | 0.86 (0.78, 0.93) | 0.54 (0.47, 0.63) |
| DOMINICAN | 1.51 (1.40, 1.65) | 1.01 (0.93, 1.12) | 1.13 (1.03, 1.25) |
| ii) Autocorrelation | | | |
| | PRICES | MONEY | INTEREST |
| COSTA RICA | 1.30 (1.21, 1.43) | 1.10 (0.99, 1.27) | 1.30 (1.07, 1.61) |
| HONDURAS | 1.14 (1.02, 1.32) | 0.91 (0.71, 1.26) | 1.39 (1.23, 1.61) |
| EL SALVADOR | 1.08 (0.96, 1.25) | 1.01 (0.87, 1.18) | 1.11 (0.98, 1.30) |
| GUATEMALA | 1.21 (1.07, 1.44) | 0.60 (0.45, 0.86) | 1.54 (1.40, 1.74) |
| NICARAGUA | 1.16 (1.03, 1.35) | 0.97 (0.86, 1.14) | 0.64 (0.47, 0.84) |
| DOMINICAN | 1.27 (1.13, 1.46) | 1.00 (0.89, 1.17) | 1.04 (0.85, 1.28) |

In bold, evidence of unit roots at the 5% level.

Next, we use a semiparametric method (Robinson, 1995) where no assumption is made on the specification of the error term, so simply,

we estimate the value of d in:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots,$$

Table 4: Estimates of d based on a semiparametric method

| PRICES | | | | | |
|-----------------------|---------------|---------------|---------------|---------------|---------------|
| | 11 | 12 | 13 | 14 | 15 |
| COSTA RICA | 1.457 | 1.466 | 1.479 | 1.500 | 1.500 |
| HONDURAS | 1.226* | 1.225* | 1.210* | 1.208* | 1.111* |
| EL SALVADOR | 1.219* | 1.209* | 1.212* | 1.152* | 1.169* |
| GUATEMALA | 1.181* | 1.219* | 1.281 | 1.318 | 1.212* |
| NICARAGUA | 1.375 | 1.444 | 1.467 | 1.330 | 1.314 |
| DOMINICAN REP. | 1.340 | 1.415 | 1.460 | 1.497 | 1.500 |
| MONEY | | | | | |
| | 11 | 12 | 13 | 14 | 15 |
| COSTA RICA | 1.411 | 1.437 | 1.467 | 1.443 | 1.202* |
| HONDURAS | 1.270 | 1.173* | 1.220* | 1.071* | 0.799 |
| EL SALVADOR | 1.170* | 1.162* | 1.202* | 1.155* | 1.143* |
| GUATEMALA | 0.619 | 0.686 | 0.624 | 0.686 | 0.697 |
| NICARAGUA | 1.307 | 1.355 | 1.191* | 1.123* | 0.962* |
| DOMINICAN REP. | 1.133* | 1.198* | 1.225* | 1.210* | 1.209* |
| INTEREST RATES | | | | | |
| | 11 | 12 | 13 | 14 | 15 |
| COSTA RICA | 0.833 | 0.939* | 1.019* | 1.028* | 1.097* |
| HONDURAS | 1.203* | 1.355 | 1.410 | 1.421 | 1.396 |
| EL SALVADOR | 1.350 | 1.230* | 1.208* | 1.249 | 1.205* |
| GUATEMALA | 1.500 | 1.500 | 1.500 | 1.500 | 1.500 |
| NICARAGUA | 1.168* | 1.178* | 1.202* | 1.115* | 1.154* |
| DOMINICAN REP. | 1.074* | 1.022* | 1.104* | 1.137* | 1.126* |
| Lower 95% I(1) | 0.752 | 0.762 | 0.771 | 0.780 | 0.787 |
| Upper 95% I(1) | 1.247 | 1.237 | 1.228 | 1.219 | 1.2125 |

*: Evidence of unit roots at the 5% level.

Next, we move to the multivariate case and examine the three series together in a fractionally cointegrated set-up, using the FCVAR approach developed by Johansen and Nielsen (2016).

Table 5: Cointegrating Rank Test with Lag length

| Rank | Log-Likelihood | LR statistic |
|---------------------------|-----------------|---------------|
| COSTA RICA | | |
| 0 | 1457.686 | 41.914 |
| 1 | 1473.913 | 9.460 |
| 2 | 1478.370 | 0.545 |
| HONDURAS | | |
| 0 | 1518.160 | 59.160 |
| 1 | 1533.548 | 29.308 |
| 2 | 1547.721 | 0.962 |
| EL SALVADOR | | |
| 0 | 1317.560 | 33.643 |
| 1 | 1330.438 | 7.889 |
| 2 | 1333.178 | 2.408 |
| GUATEMALA | | |
| 0 | 1597.933 | 84.160 |
| 1 | 1621.310 | 37.525 |
| 2 | 1639.941 | 0.264 |
| NICARAGUA | | |
| 0 | 1103.425 | 93.232 |
| 1 | 1142.264 | 15.561 |
| 2 | 1149.895 | 0.301 |
| DOMINICAN REPUBLIC | | |
| 0 | 1067.009 | 38.984 |
| 1 | 1081.310 | 10.381 |
| 2 | 1086.475 | 0.052 |

The results based on LR tests indicate that the FCVAR model outperforms the classical CVAR model in all cases.

Thus, we found evidence of fractional cointegration in all the countries of the CMCA.

Table 6: FVECMmodels**COSTA RICA**

$$\Delta^{1.164} \begin{pmatrix} \log M \\ \log R \\ \log P \end{pmatrix} - \begin{pmatrix} 13.553 \\ 3.228 \\ 3.581 \end{pmatrix} = L_{1.164} \begin{pmatrix} 0.021 \\ -0.028 \\ 0.016 \end{pmatrix} \begin{pmatrix} 1.000 \\ 0.889 \\ -1.514 \end{pmatrix} (X_t - \mu) + \Gamma \Delta^{1.164} L_{1.164} (X_t - \mu) + \varepsilon_t$$

HONDURAS

$$\Delta^{1.076} \begin{pmatrix} \log M \\ \log R \\ \log P \end{pmatrix} - \begin{pmatrix} 9.678 \\ 3.145 \\ 4.792 \end{pmatrix} = L_{1.076} \begin{pmatrix} -0.083 & -0.087 \\ -0.041 & -0.030 \\ 0.010 & -0.012 \end{pmatrix} \begin{pmatrix} 1.000 & 0.000 \\ 0.000 & 1.000 \\ -1.905 & 0.064 \end{pmatrix} (X_t - \mu) + \Gamma \Delta^{1.076} L_{1.076} (X_t - \mu) + \varepsilon_t$$

EL SALVADOR

$$\Delta^{0.937} \begin{pmatrix} \log M \\ \log R \\ \log P \end{pmatrix} - \begin{pmatrix} 7.443 \\ 2.050 \\ 4.319 \end{pmatrix} = L_{0.937} \begin{pmatrix} -0.078 \\ -0.012 \\ -0.019 \end{pmatrix} \begin{pmatrix} 1.000 \\ -0.352 \\ -1.733 \end{pmatrix} (X_t - \mu) + \Gamma \Delta^{0.937} L_{0.937} (X_t - \mu) + \varepsilon_t$$

GUATEMALA

$$\Delta^{0.889} \begin{pmatrix} \log M \\ \log R \\ \log P \end{pmatrix} - \begin{pmatrix} 9.931 \\ 2.876 \\ 4.055 \end{pmatrix} = L_{0.889} \begin{pmatrix} -0.105 & 0.015 \\ -0.061 & 0.051 \\ 0.003 & -0.042 \end{pmatrix} \begin{pmatrix} 1.000 & 0.000 \\ 0.000 & 1.000 \\ -1.567 & 0.192 \end{pmatrix} (X_t - \mu) + \Gamma \Delta^{0.889} L_{0.889} (X_t - \mu) + \varepsilon_t$$

NICARAGUA

$$\Delta^{0.886} \begin{pmatrix} \log M \\ \log R \\ \log P \end{pmatrix} - \begin{pmatrix} 9.596 \\ 3.084 \\ 4.228 \end{pmatrix} = L_{0.886} \begin{pmatrix} -0.024 \\ -0.004 \\ -0.020 \end{pmatrix} \begin{pmatrix} 1.000 \\ 0.664 \\ -1.225 \end{pmatrix} (X_t - \mu) + \Gamma_i \Delta^{0.886} L_{0.886} (X_t - \mu) + \varepsilon_t$$

DOMINICAN REPUBLIC

$$\Delta^{0.664} \begin{pmatrix} \log M \\ \log R \\ \log P \end{pmatrix} - \begin{pmatrix} 10.547 \\ 2.980 \\ 3.529 \end{pmatrix} = L_{0.664} \begin{pmatrix} -0.027 \\ 0.023 \\ 0.004 \end{pmatrix} \begin{pmatrix} 1.000 \\ -2.799 \\ -3.210 \end{pmatrix} (X_t - \mu) + \Gamma_i \Delta^{0.664} L_{0.664}^i (X_t - \mu) + \varepsilon_t$$

Table 6: Estimates of d in the FVECM model

| Country | Estimates of d |
|--------------------|----------------|
| COSTA RICA | 1.164 |
| HONDURAS | 1.076 |
| EL SALVADOR | 0.937 |
| GUATEMALA | 0.889 |
| NICARAGUA | 0.886 |
| DOMINICAN REPUBLIC | 0.664** |

** : Estimates which are significantly below 1 at the 5% level.

Table 7: FVECM Forecast Mean Square Errors:

| Country | Prices | Money | Int. Rate |
|--------------------|-----------|-----------|-----------|
| COSTA RICA | 0,0006621 | 0,0002114 | 0,0002018 |
| HONDURAS | 0,0048973 | 0,0224752 | 0,0000149 |
| EL SALVADOR | 0,0031961 | 0,0012310 | 0,0001276 |
| GUATEMALA | 0,0002791 | 0,0007938 | 0,0000298 |
| NICARAGUA | 0,0001617 | 0,0447095 | 0,0002884 |
| DOMINICAN | 0,0013325 | 0,0935417 | 0,0004660 |

5. CONCLUDING COMMENTS

Three macro variables (CPI, interest rate and monetary base) have been examined for the six countries forming the CMCA using fractional integration and cointegration techniques.

The **univariate results** indicate values of d above 1 for prices and around 1 for the interest rates and money.

In a **multivariate context**, using the FCVAR model of Nielsen and Johansen (2016), the LR tests support this specification rather than the classical CVAR, with an order of integration for the cointegrating system above 1 for Costa Rica; around 1 for Honduras and El Salvador; slightly below 1 for Guatemala and Nicaragua; and significantly below 1 for Dominican Republic.

- More work on this multivariate approach will be conducted in future papers, in particular, allowing for a more flexible specification where d and b are supposed to be different.
- Work in this direction is now in progress.

Thank you